Stringy black holes in five dimensions

Per Kraus, UCLA

with Alejandra Castro, Josh Davis and Finn Larsen.

Overview

- General comments on black holes in string theory.
- Symmetry and anomaly arguments to explain *why* micro/macro entropies agree, including corrections.
- Explicit results for 5D black holes / strings in R^2 corrected sugra. Singularity resolution, corrections to entropy.

Black holes in string theory: where are we?

- > 10 yrs since Strominger/Vafa. Many examples, intricate checks, but really just a few basic themes.
- Two classes:

  (1) exact entropy match (e.g. D1-D5-P)

  (2) matching up to numerical factors (e.g. D3-branes, $3/4$)
- What’s the distinction?
- Agreement $S_{BH} = S_{CFT}$ can seem “miraculous". Can we see agreement without computing both sides case by case?

The importance of AdS_3

- Cases with exact agreements have near horizon AdS$_3$ factor, and $1+1$ CFT dual. Agreement follows from general properties.
- Micro side: key tool is Cardy formula

$$S = 2\pi\sqrt{\frac{c_L}{6}P_L} + 2\pi\sqrt{\frac{c_R}{6}P_R} + \dots$$

  Just need $c_{L,R}$ to get large $P_{L,R}$ behavior. $c_{L,R}$ determined in vacuum.

- Gravity side: BH =BTZ = locally AdS$_3$. Same logic again leads to Cardy formula. $c_{L,R}$ again determined in vacuum.
- Can show that $c_{L,R}$ agree between two sides without having to compute them explicitly.

Anomaly inflow

- Use symmetries and anomalies to show that central charges must agree. Harvey/Minasian/Moore; P.K./Larsen
- Micro side: given D=1+1 CFT on brane worldvolume, susy relates c to R-symmetry anomaly

$$\partial_\mu j_R^\mu = k\epsilon^{\mu\nu} F_{\mu\nu} \ , \quad c = 6k$$

- Consider brane in ambient space with external gauge fields. Theory on brane is anomalous. Anomaly cancelled by inflow from bulk due to Chern-Simons terms $S_{CS} = kA \wedge dA$.
- Brane central charge determined from bulk CS term. Topological.
- Analogous argument determines c of non-susy half of CFT from gravitational anomaly.

Anomaly inflow cont.

- On gravity side assume existence of solution with same near horizon symmetries $\Rightarrow$ AdS_3.
- In asymptotic region solution looks same as before. Same CS term and same inflow, now into AdS_3 region.
- AdS_3 theory not gauge invariant by itself due to inflow across boundary. Same inflow as before. Central charge must be the same.
- Only needed to use topological terms. Robust.
- In general, central charges have classical piece plus corrections. Argument shows that they agree exactly.
- In terms of entropy, implies matching of infinite series of corrections

$$S = \frac{1}{4} A(Q,P) + \sum c_n \frac{1}{Q^n}$$

Can be computed explicitly given full set of CS terms. Also works for non-BPS / near-extremal holes.

Other corrections

- Also have corrections to Cardy formula

$$S = S_{cardy} + \sum \left(\frac{1}{P_{L,R}} \right)^n$$

Not universal: depend on details of CFT. On gravity side requires knowledge of full action and non-perturbative effects.

- OSV conjecture

$$Z_{BH}(p^I, \phi^I) = \sum_{q_I} \Omega(p^I, q_I) e^{-\phi^I q_I}$$

$$Z_{BH} = |Z_{top}|^2$$

- Would like to Laplace transform to extract degeneracies. Various ambiguities need to be resolved.

Higher derivative corrections in 4D de Wit et. al.

- Type II on $CY_3 \Rightarrow \mathcal{N} = 2$ sugra.
- Gauge fields F^I with spectrum of electric and magnetic charges (q_I, p^I) corresponding to wrapped branes. Complex moduli Y^I obeying special geometry.
- Two-deriv. theory. Consider D4-D0 black hole with charges p^I and q_0.

$$S = 2\pi\sqrt{\frac{1}{6}c_{IJK}p^I p^J p^K q_0}$$

- Infinite series of higher deriv. corrections. Subset captured by modified prepotential

$$F(Y^I) \to F(Y^I, W^2) = \sum_g F_g(Y^I)(W^2)^g$$

$F_g =$ genus g top. string free energy.

- Gives corrections

$$L = \sum_g F_g(Y^I)(C_{\mu\nu\alpha\beta})^2(F^I)^{2g-2} + \ldots$$

- Susy manifest. Look for BPS solutions. Find entropy from attractor mechanism and Wald formula.
- For large CY_3 have

$$F(Y^I, W^2) = \frac{c_{IJK} Y^I Y^J Y^K}{Y^0} + \frac{c_{2I} Y^I}{Y^0} W^2 + \ldots$$

- leads to BPS entropy.

$$S = 2\pi \sqrt{\frac{1}{6}(c_{IJK} p^I p^J p^K + c_{2I} p^I) q_0}$$

Agrees with microscopic counting from wrapped M5-branes MSW.

- Agreement to all orders in c_2. Surprising from 4D viewpoint.
- For non-BPS hole find disagreement at leading order in c_2. Top. string misses some terms at 4-deriv order and beyond Sahoo/Sen
- Small BH: $CY_3 = K3 \times T^2$. Wrap p^1 D4-branes on $K3$ with q_0 D0-branes.

$$S = 2\pi \sqrt{\frac{24}{6} p^1 q_0}$$

Corresponds to $c_L = 24 p^1$. Indeed this case is dual to p^1 wrapped heterotic strings carrying q_0 units of momentum.

5D perspective

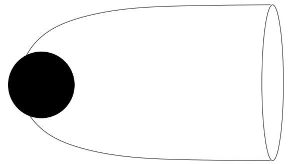
- Many aspects clearer upon lifting to M-theory on $CY_3 \times S^1$. 4D black hole becomes a black string in $5D$ with $D = 1+1$ CFT dual.
- Also have access to 5D black holes and black rings.
- Charges correspond to

$$\begin{aligned} p^I &= \mathrm{M5-branes\ on\ Ith\ 4-cycle} \\ q_I &= \mathrm{M2-branes\ on\ Ith\ 2-cycle} \\ p^0 &= \mathrm{KK-monopole} \\ q_0 &= \mathrm{momentum\ along\ intersection} \end{aligned}$$

- Taking $p^0 = 0$ but $p^I \neq 0$ gives magnetic black string with near horizon geometry $\mathrm{AdS}_3 \times S^2 \times CY_3$.
- Setting $p^I = 0$ and $q_I \neq 0$ gives electric black hole with near horizon $\mathrm{AdS}_2 \times S^3 \times CY_3$. No known micro description in general.

4D/5D connection

- Taub-NUT serves as an interpolator between 4D and 5D black holes:



- Take M-direction to be Taub-NUT fibre. Spin of 5D black hole becomes D0-charge. M2-charge become D2-charges.
- Leads to conjecture Gaiotto/Strominger/Yin

$$S_{5D}(q_I, J) = S_{4D}(q_I, q_0 = J)$$

- With OSV gives relation between 5D entropy and top. string.
- Simple relation holds at lowest order, but breaks down with higher derivs since Taub-NUT carries delocalized charges.

$$S \sim c_{2I} A^I \mathrm{Tr}(R \wedge R) \quad \Rightarrow \quad q_I^{4D} = q_I^{5D} + \frac{c_{2I}}{24}$$

5D sugra

- R^2 sugra not as well studied in 5D.
- Of special interest is mixed gauge-gravity Chern-Simons term

$$S = \int c_{2I} A^I \wedge \mathrm{Tr}(R \wedge R)$$

Coefficient known from M5-brane anomaly cancellation Duff/Liu/Minasian

- From previous arguments, susy completion should yield entropy corrections. In magnetic string case, just assuming existence of susy completion, plus near horizon AdS_3, is enough to determine entropy.
- Need full susy action to find explicit solution and find corrected entropy for 5D black hole.

5D R^2 sugra

- Action found recently by Hanaki, Ohashi, Tachikawa . Start from superconformal theory and gauge fix to Poincare.
- Weyl multiplet

$$e_\mu^a\ ,\ \ \psi_\mu\ ,\ \ V_\mu\ ,\ \ b_\mu\ ,\ \ v^{ab}\ ,\ \ \chi\ ,\ \ D$$

- Vector multiplet

$$A_\mu^I\ ,\ \ M^I\ ,\Omega^I\ ,\ \ Y^I$$

- Susy variations

$$\begin{aligned}
\delta\psi_\mu &= \left(\mathcal{D}_\mu + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}\gamma_\mu\gamma\cdot v\right)\epsilon\ , \\
\delta\Omega^I &= \left(-\frac{1}{4}\gamma\cdot F^I - \frac{1}{2}\gamma^a\partial_a M^I - \frac{1}{3}M^I\gamma\cdot v\right)\epsilon\ , \\
\delta\chi &= \left(D - 2\gamma^c\gamma^{ab}\mathcal{D}_a v_{bc} - 2\gamma^a\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(\gamma\cdot v)^2\right)\epsilon
\end{aligned}$$

5D action

- 2-deriv action

$$
\begin{aligned}
\frac{1}{2}\mathcal{L}_0 &= \partial^a \mathcal{A}_i^\alpha \partial_a \mathcal{A}_\alpha^i + \mathcal{A}^2\left(\frac{1}{8}D - \frac{3}{16}R - \frac{1}{4}v^2\right) \\
&+ \mathcal{N}\left(\frac{1}{4}D + \frac{1}{8}R + \frac{3}{2}v^2\right) + \mathcal{N}_I v^{ab} F_{ab}^I \\
&+ \mathcal{N}_{IJ}\left(\frac{1}{8}F_{ab}^I F^{Jab} + \frac{1}{4}\partial_a M^I \partial^a M^J\right) + \frac{1}{48} c_{IJK} A_a^I F_{bc}^J F_{de}^K \epsilon^{abcde} \; .
\end{aligned}
$$

Integrating out auxiliary $D, \; v^{ab}$ gives standard action

$$
\begin{aligned}
\mathcal{L}_0 &= -[-R + G_{IJ}\partial_a M^I \partial^a M^J + \frac{1}{2} G_{IJ} F_{ab}^I F^{Jab}] \\
&\quad - \frac{1}{24} c_{IJK} A_a^I F_{bc}^J F_{de}^K \epsilon^{abcde} \; , \\
G_{IJ} &= \frac{1}{2}\left(\mathcal{N}_I \mathcal{N}_J - \mathcal{N}_{IJ}\right) \\
&\mathcal{N} = \frac{1}{6} c_{IJK} M^I M^J M^K \; , \quad \mathcal{N}_I = \partial_I \mathcal{N}
\end{aligned}
$$

- D equation gives constraint $\mathcal{N} = 1$.

4-deriv action *(Hanaki, Ohashi, Tachikawa)*

$$
\begin{aligned}
\mathcal{L}_1 \;=\;& \frac{c_{2I}}{24}\Big(\frac{1}{16}\epsilon_{abcde}A^{Ia}C^{bcfg}C^{de}{}_{fg} + \frac{1}{8}M^I C^{abcd}C_{abcd} + \frac{1}{12}M^I D^2 \\
+\;& \frac{1}{6}F^{Iab}v_{ab}D + \frac{1}{3}M^I C_{abcd}v^{ab}v^{cd} + \frac{1}{2}F^{Iab}C_{abcd}v^{cd} + \frac{8}{3}M^I v_{ab}\hat{\mathcal{D}}^b\hat{\mathcal{D}}_c v^{ac} \\
+\;& \frac{4}{3}M^I\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_a v_{bc} + \frac{4}{3}M^I\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_b v_{ca} - \frac{2}{3}M^I\epsilon_{abcde}v^{ab}v^{cd}\hat{\mathcal{D}}_f v^{ef} \\
+\;& \frac{2}{3}F^{Iab}\epsilon_{abcde}v^{cd}\hat{\mathcal{D}}_f v^{ef} + F^{Iab}\epsilon_{abcde}v^c{}_f\hat{\mathcal{D}}^d v^{ef} \\
-\;& \frac{4}{3}F^{Iab}v_{ac}v^{cd}v_{db} - \frac{1}{3}F^{Iab}v_{ab}v^2 + 4M^I v_{ab}v^{bc}v_{cd}v^{da} - M^I(v^2)^2\Big)
\end{aligned}
$$

Believed to be complete set of 4-deriv terms.

- D equation now modifies special geometry constraint: $\mathcal{N} \neq 1$.
- Solving equations of motion directly obviously very difficult. But constructing BPS solutions tractable since susy variations are uncorrected.

BPS solutions

- First look for spherically symmetric solutions.
- Form Killing vector from Killing spinor

$$K^\mu = \bar{\epsilon}\gamma^\mu\epsilon$$

$K^\mu K_\mu \geq 0 \Rightarrow K^\mu$ timelike or null ($K^0 \neq 0$).

- Leads to two types of solutions

$$K^\mu = \begin{cases} \text{timelike} & \text{electric 5D BH} \;\; AdS_2 \times S^3 \\ \text{null} & \text{magnetic 5D black string} \;\; AdS_3 \times S^2 \end{cases}$$

- Ansatz

$$\begin{aligned} \text{electric}: \quad ds^2 &= e^{4U_1} dt^2 - e^{-2U_2} dx^i dx^i \\ \text{magnetic}: \quad ds^2 &= e^{2U_1}(dt^2 - dx_4^2) - e^{-4U_2} dx^i dx^i \end{aligned}$$

Spherical symmetry then fixes all fields up to radial dependence.

BPS equations

- Imposing susy fixes v, F^I and D. e.g., magnetic case

$$\begin{aligned} v &\sim e^{-2U}\partial_r U \epsilon_2 \\ F^I &\sim r^2\partial_r(M^I e^{-2U})\epsilon_2 \\ D &\sim e^{4U}\nabla^2 U \end{aligned}$$

- Maxwell equation yields M^I in terms of harmonic functions. e.g., magnetic case

$$M^I e^{-2U} = H^I = 1 + \frac{p^I}{2r}$$

- Final ingredient is constraint from D equation

$$0 = e^{-6U} + \frac{1}{6}c_{IJK}H^I H^J H^K + c_{2I}(\nabla H^I \nabla U + 2M^I \nabla^2 U)$$

Nonlinear ODE requiring numerical treatment.

Properties of solutions: asymptotic behavior

- Asymptotic behavior is a bit subtle. In 2-deriv theory solution approaches unique vacuum

$$e^{-2U} \sim 1 + \frac{c}{r^{(2)}} \; , \quad r \to \infty$$

But four derivative theory has spurious solution for small fluctuations around Minkowski

$$\delta U \sim A \sin(kr) \; , \quad k^{-1} \sim l_{Pl}$$

- Solutions are unphysical: don't show up in string spectrum. Can be removed by field redefinition:

$$U \to \tilde{U} = U + \frac{1}{k^2} U''$$

- Expect generic localized solution to match onto finite A oscillatory solution. Indeed happens. But happens far away, so not really worrisome. Same as in 4D story. Sen; Hubeny/Maloney/Rangamani

Properties of solutions: near horizon

- Structure of near horizon region understood from enhanced susy

$$\delta(\text{fermion}) = (\text{stuff} = 0)\epsilon$$

Forces geometry to be AdS$_{2,3} \times S^{3,2}$ and moduli to take attractor values.

- Find

$$\text{magnetic}: \qquad \ell_A = 2\ell_S = (\frac{1}{6}c_{IJK}p^I p^J p^K + \frac{1}{12}c_{2I}p^I)^{1/3}$$
$$\text{electric}: \qquad \ell_S = 2\ell_A = (\frac{1}{6}c_{IJK}\hat{M}^I\hat{M}^J\hat{M}^K - \frac{1}{12}c_{2I}p^I)^{1/3}$$

with

$$\frac{1}{2}c_{IJK}\hat{M}^J\hat{M}^K = q_I + \frac{1}{8}c_{2I}$$

- Exhibits resolution of naked singularity. Setting $cp^3 = 0$ but $c_{2I}p^I \neq 0$ gives smooth would-be singular solution.

Entropy

- In presence of higher derivatives area law is replaced by Wald entropy in general

$$S \sim \int_{hor} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\alpha\beta}} \epsilon_{\mu\nu} \epsilon_{\alpha\beta}$$

  Can be tedious to evaluate.

- Major simplifications for black holes with near horizon AdS. Can use extremization approach.

- Magnetic AdS$_3$ case: have predictions for corrections to $c_{L,R}$ from anomaly argument. Can now check explicitly.

- c-extremization: given any action with near horizon AdS$_3 \times S^2$, define

$$c = -6\ell_A^3 \ell_s^2 \mathcal{L}$$

  Regard as function of unknown $\ell_{A,S}$ and other fields, but at fixed charge. Equations of motion reduce to extremizing c.

c-extremization cont.

- At extremum

$$c = \frac{1}{2}(c_L + c_R)$$

Show by relating CFT trace anomaly $T^\mu_\mu = \frac{c}{24} R$ to divergent part of bulk action.

- Evaluation of on-shell action indeed reproduces result predicted from anomalies.
- Corrected entropy now follows form Cardy formula
- Further prediction: any additional higher derivative terms will have to cancel out in c-function.

Entropy of electric solutions

- Given AdS_2, most convenient to use Sen's entropy function. Extremize

$$S = \pi \ell_A^2 \ell_S^3 (F^I_{tr} \frac{\partial \mathcal{L}}{\partial F^I_{tr}} - \mathcal{L})$$

- 2-deriv result

$$S = 2\pi \ell_S^3 = 2\pi (\frac{1}{6} c_{IJK} \hat{M}^I \hat{M}^J \hat{M}^K)$$

  with $\frac{1}{2} c_{IJK} \hat{M}^J \hat{M}^K = q_I$.

- Modifications due to 4-deriv terms are surprisingly simple. Geometrically,

$$S = 2\pi \mathcal{N} \ell_s^3 \, , \quad \mathcal{N} = 1 + \frac{c_{2I} M^I}{12 \ell_s^2}$$

  In 2-deriv theory $\mathcal{N} = 1$ corresponds to volume of CY_3. Assuming same here (ambiguous), still have $S = \frac{A}{4}$.

- In terms of charges, only change is shift in charges

$$q_I \rightarrow q_I + \frac{1}{8} c_{2I}$$

- Strictly valid to first order in c_2.

Comparison to previous results

- Vafa computed microscopic corrections to entropy of electric BH on elliptically fibred CY_3. Gets same leading order correction as we find

$$q \cdot q \to q \cdot q + \frac{1}{4} c_2 \cdot q$$

- Huang et. al. recently numerically computed asymptotic M2 degeneracy from GV invariants. Found agreement with $\frac{1}{8} c_{2I}$ shift.

- Top-string and 4D$\to$ 5D lift gives same entropy expressed in terms of $\hat{M}^I$, but with a different charge shift Strominger et. al

$$q_I \to q_I + \frac{1}{6} c_{2I}$$

- To resolve this we constructed black holes on Taub-NUT. Only change is in Gauss' law

$$\nabla \cdot E^I = \frac{c_{2I}}{8 \cdot 24} (R_{ijkl} R^{ijkl})_{TN}$$

Delocalized charge $q_I = \frac{c_{2I}}{24}$ accounts for different shifts.

Spinning black holes

- Can find higher derivative BMPV solution. Attractor moduli now depend on angular momentum $(\hat{j} = J/\ell_s^3)$

$$\frac{1}{2}c_{IJK}\hat{M}^J\hat{M}^K = q_I + c_{2I}\left(\frac{1}{8} - \frac{1}{6}\hat{j}^2\right)$$

and the entropy is

$$S = 2\pi\sqrt{1-\hat{j}^2}\left(\frac{1}{6}c_{IJK}\hat{M}^I\hat{M}^J\hat{M}^K + \frac{1}{6}\hat{j}^2 c_{2I}\hat{M}^I\right)$$

- Naive result using 4D/5D now off by

$$q_0 = J + \frac{1}{12}c_{2I}M^I\frac{e^0}{(1+(e^0)^2)}$$

Black rings

- Full R^2 black ring geometry not found yet.
- Entropy can be computed in terms of near horizon data. Near horizon geometry governed by same attractor as magnetic string.

$$S = 2\pi \frac{(p^3 + \frac{1}{6} c_{2I} p^I)}{e^0}$$

- Can also write this as

$$S = (2 - \mathcal{N}) \frac{A}{4G_5}$$

- e^0 is a near horizon quantity. Need to trade it for ring charges.

Conclusions

- 5D higher derivative corrections under good control. Should help in pinning down microscopic description of 5D black holes on generic CY_3, and of black rings.
- To do: full black ring solutions, non-BPS solutions, ...