Stringy black holes in five dimensions

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Overview

• General comments on black holes in string theory.

• Symmetry and anomaly arguments to explain why micro/macro entropies agree, including corrections.

• Explicit results for 5D black holes / strings in $R^2$ corrected sugra. Singularity resolution, corrections to entropy.
Black holes in string theory: where are we?

• > 10 yrs since Strominger/Vafa. Many examples, intricate checks, but really just a few basic themes.

• Two classes:
  (1) exact entropy match (e.g. D1-D5-P)
  (2) matching up to numerical factors (e.g. D3-branes, $3/4$)

• What’s the distinction?

• Agreement $S_{BH} = S_{CFT}$ can seem “miraculous". Can we see agreement without computing both sides case by case?
The importance of AdS$_3$

- Cases with exact agreements have near horizon AdS$_3$ factor, and $1 + 1$ CFT dual. Agreement follows from general properties.

- Micro side: key tool is Cardy formula

$$S = 2\pi \sqrt{\frac{c_L}{6} P_L} + 2\pi \sqrt{\frac{c_R}{6} P_R} + \ldots$$

Just need $c_{L,R}$ to get large $P_{L,R}$ behavior. $c_{L,R}$ determined in vacuum.

- Gravity side: BH = BTZ = locally AdS$_3$. Same logic again leads to Cardy formula. $c_{L,R}$ again determined in vacuum.

- Can show that $c_{L,R}$ agree between two sides without having to compute them explicitly.
Anomaly inflow

• Use symmetries and anomalies to show that central charges must agree. Harvey/Minasian/Moore; P.K./Larsen

• Micro side: given D=1+1 CFT on brane worldvolume, susy relates $c$ to R-symmetry anomaly

\[ \partial_\mu j^\mu_R = k \epsilon^{\mu\nu} F_{\mu\nu} , \quad c = 6k \]

• Consider brane in ambient space with external gauge fields. Theory on brane is anomalous. Anomaly cancelled by inflow from bulk due to Chern-Simons terms $S_{CS} = kA \wedge dA$.

• Brane central charge determined from bulk CS term. Topological.

• Analogous argument determines $c$ of non-susy half of CFT from gravitational anomaly.
**Anomaly inflow cont.**

- On gravity side assume existence of solution with same near horizon symmetries $\Rightarrow \text{AdS}_3$.

- In asymptotic region solution looks same as before. Same CS term and same inflow, now into AdS$_3$ region.

- AdS$_3$ theory not gauge invariant by itself due to inflow across boundary. Same inflow as before. Central charge must be the same.

- Only needed to use topological terms. Robust.

- In general, central charges have classical piece plus corrections. Argument shows that they agree exactly.

- In terms of entropy, implies matching of infinite series of corrections

  \[
  S = \frac{1}{4} A(Q, P) + \sum c_n \frac{1}{Q^n}
  \]

  Can be computed explicitly given full set of CS terms. Also works for non-BPS / near-extremal holes.
Other corrections

• Also have corrections to Cardy formula

\[ S = S_{\text{cardy}} + \sum \left( \frac{1}{P_{L,R}} \right)^n \]

Not universal: depend on details of CFT. On gravity side requires knowledge of full action and non-perturbative effects.

• OSV conjecture

\[ Z_{BH}(p^I, \phi^I) = \sum_{q_I} \Omega(p^I, q_I)e^{-\phi^I q_I} \]

\[ Z_{BH} = |Z_{top}|^2 \]

• Would like to Laplace transform to extract degeneracies. Various ambiguities need to be resolved.
Higher derivative corrections in 4D

devit et. al.

- Type II on $CY_3 \Rightarrow \mathcal{N} = 2$ sugra.

- Gauge fields $F^I$ with spectrum of electric and magnetic charges $(q_I, p^I)$ corresponding to wrapped branes. Complex moduli $Y^I$ obeying special geometry.

- Two-deriv. theory. Consider D4-D0 black hole with charges $p^I$ and $q_0$.

$$S = 2\pi \sqrt{\frac{1}{6} c_{IKI} p^I p^J p^K q_0}$$

- Infinite series of higher deriv. corrections. Subset captured by modified prepotential

$$F(Y^I) \rightarrow F(Y^I, W^2) = \sum_g F_g(Y^I)(W^2)^g$$

$F_g = \text{genus } g \text{ top. string free energy.}$

- Gives corrections

$$L = \sum_g F_g(Y^I)(C_{\mu\nu\alpha\beta})^2(F^I)^{2g-2} + \ldots$$
• Susy manifest. Look for BPS solutions. Find entropy from attractor mechanism and Wald formula.

• For large $C_{Y3}$ have

$$F(Y^I, W^2) = \frac{c_{IJK} Y^I Y^J Y^K}{Y^0} + \frac{c_{2I} Y^I}{Y^0} W^2 + \ldots$$

• leads to BPS entropy.

$$S = 2\pi \sqrt{\frac{1}{6} (c_{IJK} p^I p^J p^K + c_{2I} p^I)} q_0$$

Agrees with microscopic counting from wrapped M5-branes MSW.

• Agreement to all orders in $c_2$. Surprising from 4D viewpoint.

• For non-BPS hole find disagreement at leading order in $c_2$. Top. string misses some terms at 4-deriv order and beyond Sahoo/Sen

• Small BH: $C_{Y3} = K3 \times T^2$. Wrap $p^1$ D4-branes on $K3$ with $q_0$ D0-branes.

$$S = 2\pi \sqrt{\frac{24}{6} p^1 q_0}$$

Corresponds to $c_L = 24 p^1$. Indeed this case is dual to $p^1$ wrapped heterotic strings carrying $q_0$ units of momentum.
**5D perspective**

- Many aspects clearer upon lifting to M-theory on $CY_3 \times S^1$. 4D black hole becomes a black string in 5$D$ with $D = 1 + 1$ CFT dual.

- Also have access to 5D black holes and black rings.

- Charges correspond to

  \[
  \begin{align*}
  p^I &= \text{M5 \text{ -- branes on Ith 4 \text{ -- cycle}}} \\
  q_I &= \text{M2 \text{ -- branes on Ith 2 \text{ -- cycle}}} \\
  p^0 &= \text{KK \text{ -- monopole}} \\
  q_0 &= \text{momentum along intersection}
  \end{align*}
  \]

- Taking $p^0 = 0$ but $p^I \neq 0$ gives magnetic black string with near horizon geometry $AdS_3 \times S^2 \times CY_3$.

- Setting $p^I = 0$ and $q_I \neq 0$ gives electric black hole with near horizon $AdS_2 \times S^3 \times CY_3$. No known micro description in general.
4D/5D connection

- Taub-NUT serves as an interpolator between 4D and 5D black holes:

- Take M-direction to be Taub-NUT fibre. Spin of 5D black hole becomes D0-charge. M2-charge become D2-charges.

- Leads to conjecture Gaiotto/Strominger/Yin

\[ S_{5D}(q_I, J) = S_{4D}(q_I, q_0 = J) \]

- With OSV gives relation between 5D entropy and top. string.

- Simple relation holds at lowest order, but breaks down with higher derivs since Taub-NUT carries delocalized charges.

\[ S \sim c_{2I} A^I \text{Tr}(R \wedge R) \quad \Rightarrow \quad q^{4D}_I = q^{5D}_I + \frac{c_{2I}}{24} \]
5D sugra

- $R^2$ sugra not as well studied in 5D.
- Of special interest is mixed gauge-gravity Chern-Simons term

$$ S = \int c_{2I} A^I \wedge \text{Tr}(R \wedge R) $$

Coefficient known from M5-brane anomaly cancellation Duff/Liu/Minasian

- From previous arguments, susy completion should yield entropy corrections. In magnetic string case, just assuming existence of susy completion, plus near horizon AdS$_3$, is enough to determine entropy.
- Need full susy action to find explicit solution and find corrected entropy for 5D black hole.
**5D $R^2$ sugra**

- Action found recently by Hanaki, Ohashi, Tachikawa. Start from superconformal theory and gauge fix to Poincare.

- Weyl multiplet

\[ e^a_\mu, \, \psi_\mu, \, V_\mu, \, b_\mu, \, v^{ab}, \, \chi, \, D \]

- Vector multiplet

\[ A^I_\mu, \, M^I, \, \Omega^I, \, Y^I \]

- Susy variations

\[
\begin{align*}
\delta \psi_\mu &= \left( D_\mu + \frac{1}{2} v^{ab} \gamma_{\mu ab} - \frac{1}{3} \gamma_\mu \gamma \cdot v \right) \epsilon, \\
\delta \Omega^I &= \left( -\frac{1}{4} \gamma \cdot F^I - \frac{1}{2} \gamma^a \partial_a M^I - \frac{1}{3} M^I \gamma \cdot v \right) \epsilon, \\
\delta \chi &= \left( D - 2 \gamma^c \gamma^{ab} D_a v_{bc} - 2 \gamma^a \epsilon_{abcde} v^{bc} v^{de} + \frac{4}{3} (\gamma \cdot v)^2 \right) \epsilon
\end{align*}
\]
5D action

- 2-deriv action

\[
\frac{1}{2} \mathcal{L}_0 = \partial^a A^\alpha_i \partial_a A^i_\alpha + A^2 \left( \frac{1}{8} D - \frac{3}{16} R - \frac{1}{4} v^2 \right) \\
+ \mathcal{N} \left( \frac{1}{4} D + \frac{1}{8} R + \frac{3}{2} v^2 \right) + \mathcal{N} I v^{ab} F^I_{ab} \\
+ \mathcal{N}_{IJ} \left( \frac{1}{8} F^I_{ab} F^{Jab} + \frac{1}{4} \partial_a M^I \partial^a M^J \right) + \frac{1}{48} c_{IJK} A^I_a F^J_{bc} F^K_{de} \epsilon^{abcde}.
\]

Integrating out auxiliary $D$, $v^{ab}$ gives standard action

\[
\mathcal{L}_0 = -\left[ -R + G_{IJ} \partial_a M^I \partial^a M^J + \frac{1}{2} G_{IJ} F^I_{ab} F^{Jab} \right] \\
- \frac{1}{24} c_{IJK} A^I_a F^J_{bc} F^K_{de} \epsilon^{abcde},
\]

\[
G_{IJ} = \frac{1}{2} \left( \mathcal{N}_I \mathcal{N}_J - \mathcal{N}_{IJ} \right)
\]

\[
\mathcal{N} = \frac{1}{6} c_{IJK} M^I M^J M^K, \quad \mathcal{N}_I = \partial_I \mathcal{N}
\]

- $D$ equation gives constraint $\mathcal{N} = 1$. 
\( \mathcal{L}_1 = \frac{c_2 I}{24} \left( \frac{1}{16} \epsilon_{abcde} A^{Ia} C^{bcfg} C_{fg}^{de} + \frac{1}{8} M^I C^{abcd} C_{abcd} + \frac{1}{12} M^I D^2 \right) 
+ \frac{1}{6} F^{Iab} v_{ab} D + \frac{1}{3} M^I C_{abcd} v^{ab} v^{cd} + \frac{1}{2} F^{Iab} C_{abcd} v^{cd} + \frac{8}{3} M^I v_{ab} \hat{D}^b \hat{D}^c v^{ac} 
+ \frac{4}{3} M^I \hat{D}^a v^{bc} \hat{D}_a v_{bc} + \frac{4}{3} M^I \hat{D}^a v^{bc} \hat{D}_b v_{ca} - \frac{2}{3} M^I \epsilon_{abcde} v^{ab} v^{cd} \hat{D}_f v^{ef} 
+ \frac{2}{3} F^{Iab} \epsilon_{abcde} v^{cd} \hat{D}_f v^{ef} + F^{Iab} \epsilon_{abcde} v^c_f \hat{D}^d v^{ef} 
- \frac{4}{3} F^{Iab} v_{ac} v^{cd} v_{db} - \frac{1}{3} F^{Iab} v_{ab} v^2 + 4 M^I v_{ab} v^{bc} v_{cd} v^{da} - M^I (v^2)^2 \) 

Believed to be complete set of 4-deriv terms.

- \( D \) equation now modifies special geometry constraint: \( \mathcal{N} \neq 1 \).
- Solving equations of motion directly obviously very difficult. But constructing BPS solutions tractable since susy variations are uncorrected.
**BPS solutions**

- First look for spherically symmetric solutions.

- Form Killing vector from Killing spinor

\[ K^\mu = \bar{\epsilon} \gamma^\mu \epsilon \]

\[ K^\mu K_\mu \geq 0 \Rightarrow K^\mu \text{ timelike or null (} K^0 \neq 0) . \]

- Leads to two types of solutions

\[ K^\mu = \begin{cases} 
\text{timelike} & \text{electric 5D BH } AdS_2 \times S^3 \\
\text{null} & \text{magnetic 5D black string } AdS_3 \times S^2 
\end{cases} \]

- Ansatz

  electric :  \[ ds^2 = e^{4U_1} dt^2 - e^{-2U_2} dx^i dx^i \]

  magnetic :  \[ ds^2 = e^{2U_1} (dt^2 - dx_4^2) - e^{-4U_2} dx^i dx^i \]

Spherical symmetry then fixes all fields up to radial dependence.
BPS equations

- Imposing susy fixes $v, F^I$ and $D$. e.g., magnetic case

\[ v \sim e^{-2U} \partial_r U \epsilon_2 \]
\[ F^I \sim r^2 \partial_r (M^I e^{-2U}) \epsilon_2 \]
\[ D \sim e^{4U} \nabla^2 U \]

- Maxwell equation yields $M^I$ in terms of harmonic functions. e.g., magnetic case

\[ M^I e^{-2U} = H^I = 1 + \frac{p^I}{2r} \]

- Final ingredient is constraint from $D$ equation

\[ 0 = e^{-6U} + \frac{1}{6} c_{IJK} H^I H^J H^K + c_{2I} (\nabla H^I \nabla U + 2 M^I \nabla^2 U) \]

Nonlinear ODE requiring numerical treatment.
Properties of solutions: asymptotic behavior

- Asymptotic behavior is a bit subtle. In 2-deriv theory solution approaches unique vacuum

\[ e^{-2U} \sim 1 + \frac{c}{r^{(2)}}, \quad r \to \infty \]

But four derivative theory has spurious solution for small fluctuations around Minkowski

\[ \delta U \sim A \sin(kr), \quad k^{-1} \sim l_P, \]

- Solutions are unphysical: don’t show up in string spectrum. Can be removed by field redefinition:

\[ U \to \tilde{U} = U + \frac{1}{k^2} U^{''} \]

- Expect generic localized solution to match onto finite oscillatory solution. Indeed happens. But happens far away, so not really worrisome. Same as in 4D story. Sen; Hubeny/Maloney/Rangamani
Properties of solutions: near horizon

- Structure of near horizon region understood from enhanced susy

\[
\delta(\text{fermion}) = (\text{stuff} = 0) \epsilon
\]

Forces geometry to be \( \text{AdS}_{2,3} \times S^{3,2} \) and moduli to take attractor values.

- Find

\[
\begin{align*}
l_A &= 2l_S = \left( \frac{1}{6} c_{IJK} p^I p^J p^K + \frac{1}{12} c_{2IP}^I \right)^{1/3} \\
l_S &= 2l_A = \left( \frac{1}{6} c_{IJK} \hat{M}^I \hat{M}^J \hat{M}^K - \frac{1}{12} c_{2IP}^I \right)^{1/3}
\end{align*}
\]

with

\[
\frac{1}{2} c_{IJK} \hat{M}^J \hat{M}^K = q_I + \frac{1}{8} c_{2I}
\]

- Exhibits resolution of naked singularity. Setting \( cp^3 = 0 \) but \( c_{2IP}^I \neq 0 \) gives smooth would-be singular solution.
Entropy

- In presence of higher derivatives area law is replaced by Wald entropy in general

\[ S \sim \int_{\text{hor}} \frac{\delta L}{\delta R_{\mu \nu \alpha \beta}} \epsilon_{\mu \nu} \epsilon_{\alpha \beta} \]

Can be tedious to evaluate.

- Major simplifications for black holes with near horizon AdS. Can use extremization approach.

- Magnetic AdS\(_3\) case: have predictions for corrections to \(c_{L,R}\) from anomaly argument. Can now check explicitly.

- \(c\)-extremization: given any action with near horizon AdS\(_3 \times S^2\), define

\[ c = -6 \ell_A^3 \ell_s^2 \mathcal{L} \]

Regard as function of unknown \(\ell_{A,S}\) and other fields, but at fixed charge. Equations of motion reduce to extremizing \(c\).
c-extremization cont.

• At extremum

\[ c = \frac{1}{2} (c_L + c_R) \]

Show by relating CFT trace anomaly \( T_\mu^\mu = \frac{c}{24} R \) to divergent part of bulk action.

• Evaluation of on-shell action indeed reproduces result predicted from anomalies.

• Corrected entropy now follows form Cardy formula

• Further prediction: any additional higher derivative terms will have to cancel out in c-function.
Entropy of electric solutions

• Given AdS$_2$, most convenient to use Sen’s entropy function. Extremize

\[ S = \pi \ell_A \ell_S^3 (F_{tr}^{I} \frac{\partial \mathcal{L}}{\partial F_{tr}^{I}} - \mathcal{L}) \]

• 2-deriv result

\[ S = 2\pi \ell_S^3 = 2\pi \left( \frac{1}{6} c_{IJK} \hat{M}^I \hat{M}^J \hat{M}^K \right) \]

with \( \frac{1}{2} c_{IJK} \hat{M}^J \hat{M}^K = q_I \).

• Modifications due to 4-deriv terms are surprisingly simple. Geometrically,

\[ S = 2\pi \mathcal{N} \ell_S^3, \quad \mathcal{N} = 1 + \frac{c_{2I} M^I}{12 \ell_s^2} \]

In 2-deriv theory \( \mathcal{N} = 1 \) corresponds to volume of \( CY_3 \). Assuming same here (ambiguous), still have \( S = \frac{A}{4} \).

• In terms of charges, only change is shift in charges

\[ q_I \rightarrow q_I + \frac{1}{8} c_{2I} \]

• Strictly valid to first order in \( c_2 \).
Comparison to previous results

- Vafa computed microscopic corrections to entropy of electric BH on elliptically fibred $CY_3$. Gets same leading order correction as we find

$$q \cdot q \rightarrow q \cdot q + \frac{1}{4} c_2 \cdot q$$

- Huang et. al. recently numerically computed asymptotic M2 degeneracy from GV invariants. Found agreement with $\frac{1}{8} c_2 I$ shift.

- Top-string and 4D→ 5D lift gives same entropy expressed in terms of $\hat{M}^I$, but with a different charge shift Strominger et. al

$$q_I \rightarrow q_I + \frac{1}{6} c_2 I$$

- To resolve this we constructed black holes on Taub-NUT. Only change is in Gauss’ law

$$\nabla \cdot E^I = \frac{c_2 I}{8 \cdot 24} (R_{ijkl} R^{ijkl})_{TN}$$

Delocalized charge $q_I = \frac{c_2 I}{24}$ accounts for different shifts.
Spinning black holes

- Can find higher derivative BMPV solution. Attractor moduli now depend on angular momentum \((\hat{j} = J/\ell^3_s)\)

\[
\frac{1}{2} c_{IJK} \hat{M}^J \hat{M}^K = q_I + c_{2I} \left( \frac{1}{8} - \frac{1}{6} \hat{j}^2 \right)
\]

and the entropy is

\[
S = 2\pi \sqrt{1 - \hat{j}^2} \left( \frac{1}{6} c_{IJK} \hat{M}^I \hat{M}^J \hat{M}^K + \frac{1}{6} \hat{j}^2 c_{2I} \hat{M}^I \right)
\]

- Naive result using 4D/5D now off by

\[
q_0 = J + \frac{1}{12} c_{2I} M^I \left( \frac{e^0}{1 + (e^0)^2} \right)
\]
Black rings

- Full $R^2$ black ring geometry not found yet.
- Entropy can be computed in terms of near horizon data. Near horizon geometry governed by same attractor as magnetic string.

$$S = 2\pi \frac{\left(p^3 + \frac{1}{6} c_{2I} p^I \right)}{e^0}$$

- Can also write this as

$$S = (2 - \mathcal{N}) \frac{A}{4G_5}$$

- $e^0$ is a near horizon quantity. Need to trade it for ring charges.
Conclusions

- 5D higher derivative corrections under good control. Should help in pinning down microscopic description of 5D black holes on generic $CY_3$, and of black rings.

- To do: full black ring solutions, non-BPS solutions, ...