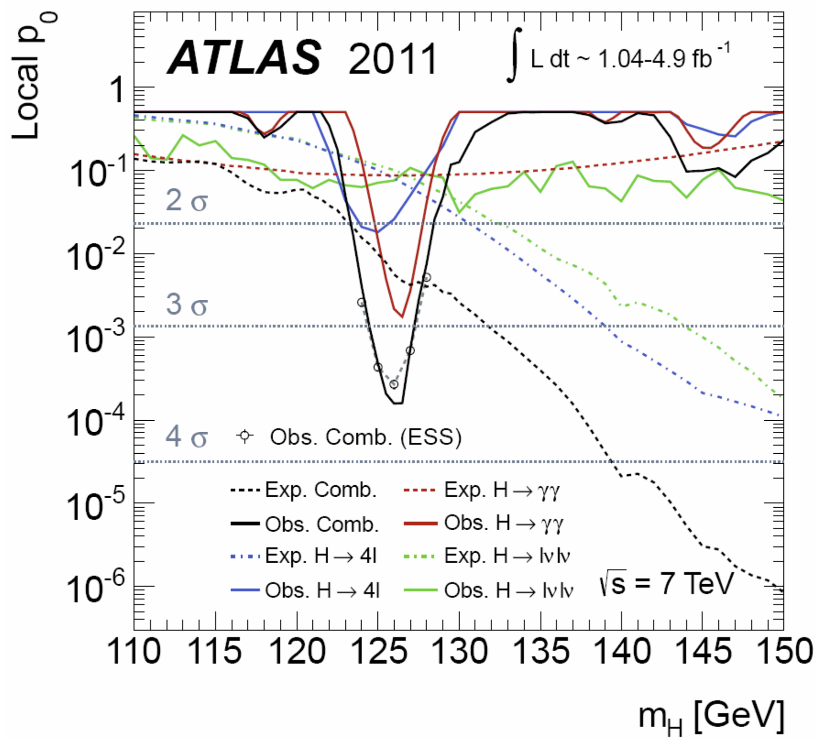
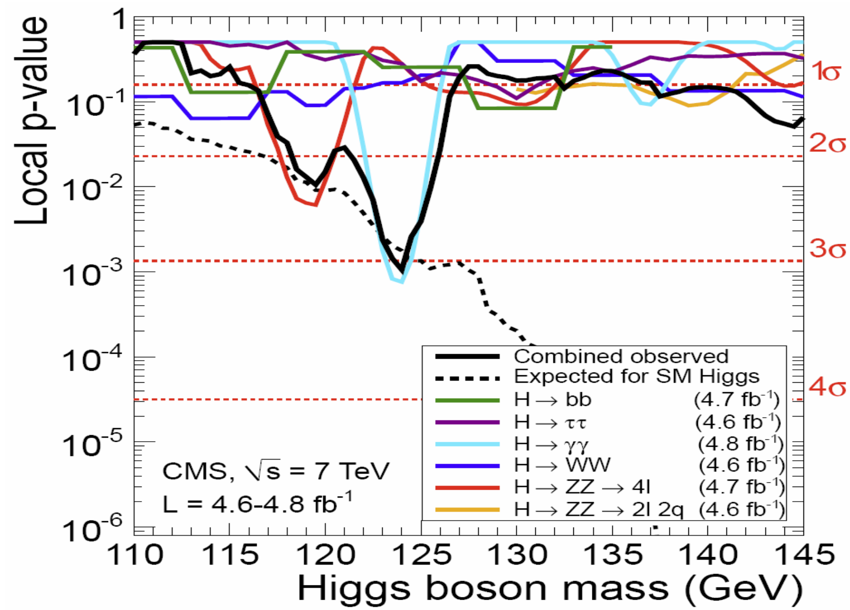
NATURALNESS AND MINIMAL SUPERSYMMETRY

Jonathan Feng, UC Irvine

UC Berkeley Particle Seminar, 13 February 2012

HIGGS BOSONS AT LHC

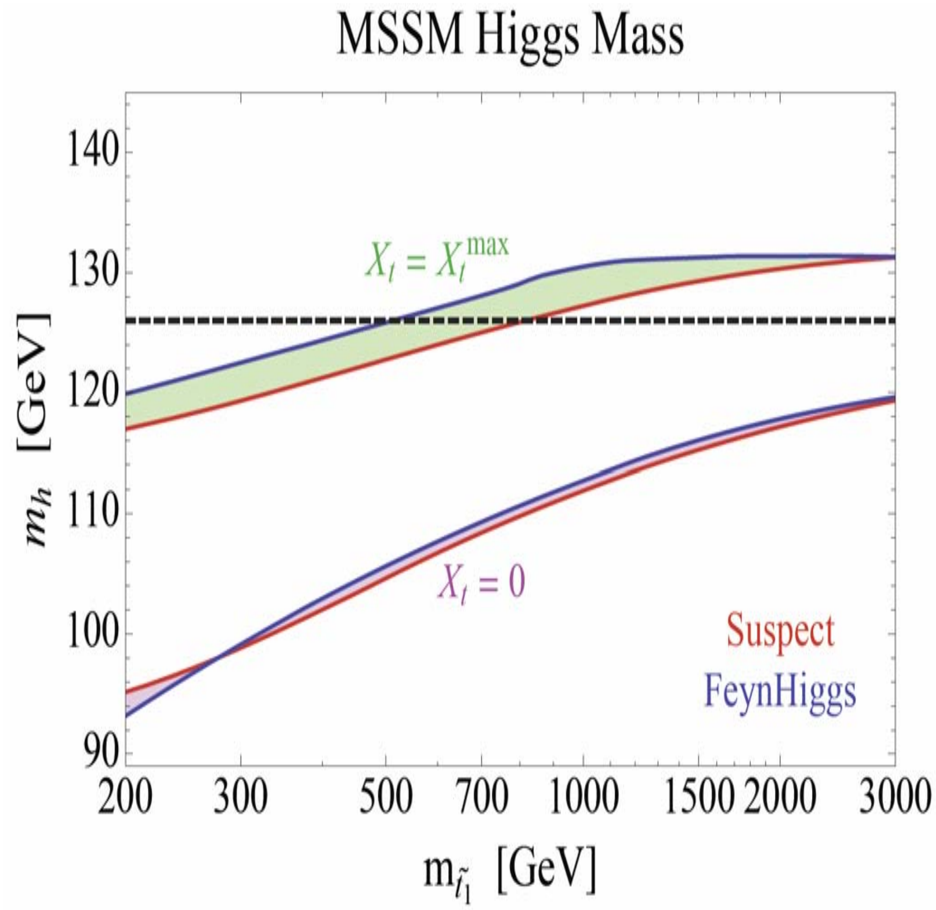




- Light Higgs excluded outside 115.5 GeV < m_H < 127 GeV
- Hints for Higgs signal in the upper half of this interval
- No strong indications of non-SM Higgs couplings

HIGGS RESULTS AND SUSY

- 30,000 foot view: great for SUSY
- Closer view: challenging for SUSY
  - Higgs mass requires heavy top squarks
  - Naturalness requires light top squarks
- This tension is much more direct that the tension created by bounds on flavor and CP violation
- It has been present (to a lesser degree) since LEP2



Hall, Pinner, Ruderman (2011)

OUTLINE

- Naturalness

- Focus Point SUSY (Gravity-Mediated SUSY)

  Work with Matchev, Moroi, Wilczek, Cheng, Polonsky (1998-2000)
  Feng, Matchev, Sanford (2011, in progress)

- Goldilocks SUSY (Gauge-Mediated SUSY)

  Work with Rajaraman, Takayama, Smith, Cembranos (2003-2007)
  Feng, Surujon, Yu (in progress)

NATURALNESS

- Two approaches:
- Option 1: "I know it when I see it." Justice Potter Stewart
- Option 2: Quantify with some well-defined naturalness prescription
- Option 1 acknowledges that naturalness is subjective, but is a non-starter. Option 2 provides an opportunity for discussion and insights, as long as its limitations are appreciated.

A NATURALNESS PRESCRIPTION

- Step 1: Choose a framework with input parameters. E.g., mSUGRA with

$$\{P_{\text{input}}\} = \{m_0, M_{1/2}, A_0, \tan\beta, \text{sign}(\mu)\}$$

- Step 2: Fix all remaining parameters with RGEs, low energy constraints. E.g., at the weak scale, tree-level,

$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

- Step 3: Choose a set of parameters as free, independent, and fundamental. E.g., mSUGRA with

$$\{a_i\} = \{m_0, M_{1/2}, A_0, B_0, \mu_0\}$$

- Step 4: Define sensitivity parameters

$$c_i \equiv \left| \frac{\partial \ln m_Z}{\partial \ln a_i} \right|$$

Ellis, Enqvist, Nanopoulos, Zwirner (1986)
Barbieri, Giudice (1988)

- Step 5: Define the fine-tuning parameter

$$c = \max\{c_i\}$$

COMMENTS

- Step 1: Choose a framework with input parameters. E.g., mSUGRA with

$$\{P_{\text{input}}\} = \{m_0, M_{1/2}, A_0, \tan\beta, \text{sign}(\mu)\}$$

This is absolutely crucial. Generic SUSY-breaking is excluded, there must be structure leading to correlated parameters, and the correlations impact naturalness. There is no model-independent measure of naturalness.

- Step 2: Fix all remaining parameters with RGEs, low energy constraints. E.g., at the weak scale

$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

Important to refine this to include 2-loop RGEs, 1-loop threshold corrections, minimize the potential at some appropriate scale (typically, the geometric mean of stop masses).

COMMENTS

- Step 3: Choose a set of parameters as free, independent, and fundamental. E.g., mSUGRA with $\{a_i\} = \{m_0, M_{1/2}, A_0, B_0, \mu_0\}$

  A popular choice is $\{a_i\} = \{\mu_0\}$, which leads to $c = 2\mu^2/m_Z^2$. This is a simple, but completely deficient and misleading, measure of naturalness.

  Should we include other parameters, like y_t?
  - No – Ellis, Enqvist, Nanopoulos, Zwirner (1986); Ciafaloni, Strumia (1996), Bhattacharyya, Romanino (1996); Chan, Chattopadhyay, Nath (1997); Barbieri, Strumia (1998); Giusti, Romanino, Strumia (1998); Chankowski, Ellis, Olechowski, Pokorski (1998); …
  - Yes – Barbieri, Giudice (1988); Ross, Roberts (1992); de Carlos, Casas (1993); Anderson, Castano (1994); Romanino, Strumia (1999); …

  We favor No – we are trying understand the naturalness of the SUSY explanation of the gauge hierarchy, so include only SUSY breaking parameters. Note: this is not an issue of what is measured and what isn't: with our current understanding, if μ were measured to be 1 EeV ± 1 eV, it will be precisely measured, but completely unnatural.

COMMENTS

- Step 4: Define sensitivity parameters $c_i \equiv \left| \frac{\partial \ln m_Z}{\partial \ln a_i} \right|$.

Ellis, Enqvist, Nanopoulos, Zwirner (1986)
Barbieri, Giudice (1988)

Why not $c_i \equiv \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right|$ (original definition) or $c_i \equiv \left| \frac{\partial \ln m_Z}{\partial \ln a_i^2} \right|$?

Factors of 2 or 4 are completely insignificant.

- Step 5: Define the fine-tuning parameter $c = \max\{c_i\}$.

Why not add in quadrature? What if c is large for all possible parameter choices (cf. Λ_{QCD}).?

De Carlos, Casas (1993); Anderson, Castano (1994)

And finally, what is the maximal natural value for c – 10, 100, 1000, … ? If SUSY reduces c from 10^{32} to 1000, isn't that enough?

GENERAL STRATEGIES

- Hidden Higgs, Buried Higgs: Make m_h < 115 GeV compatible with collider constraints

Dermisek, Gunion (2005); Bellazzini, Csaki, Falkowski, Weiler (2009); …

- Golden region, mirage mediation: Lower the messenger scale to the weak scale, generate large stop mixing

Kitano, Nomura (2005); Perelstein, Spethmann (2007)…

- Beyond the MSSM (NMSSM,…): Increase particle content to raise m_h naturally, accommodate non-SM Higgs properties

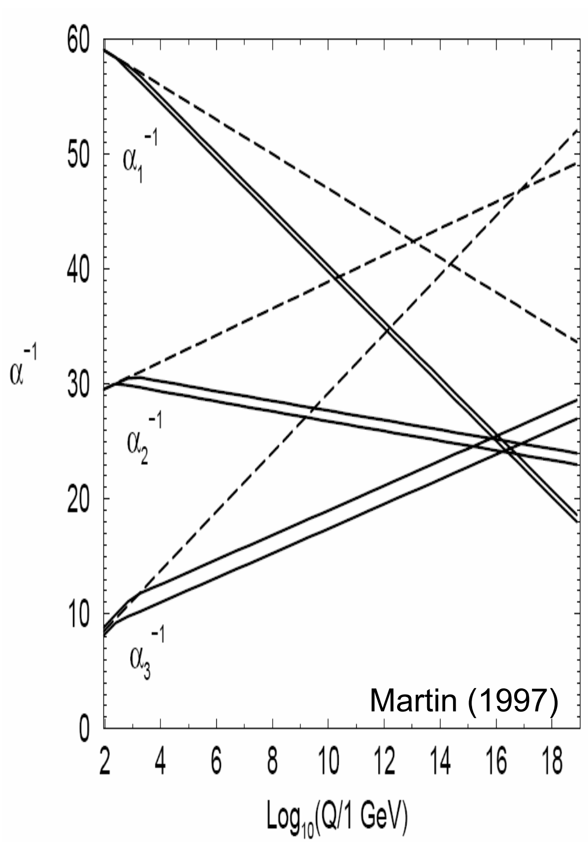
Hall, Pinner, Ruderman (2011); Ellwanger (2011); Arvanitaki, Villadoro (2011); Gunion, Jiang, Kraml (2011); Perez (2012); King, Muhlleitner, Nevzorov (2012); Kang, Li, Li (2012);…

- Focus Point SUSY: Dynamically generated naturalness

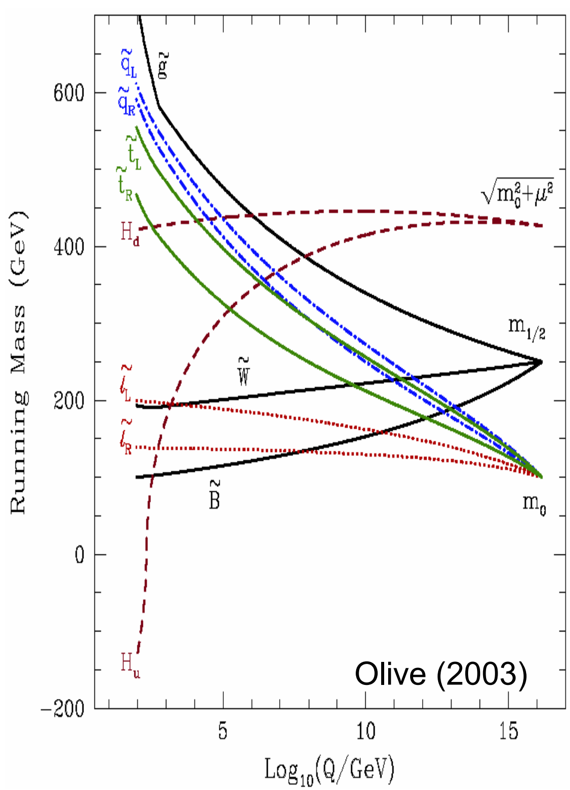
Feng, Matchev, Moroi (1999); Feng, Matchev, Wilczek (2000); Feng, Matchev (2000); Abe, Kobayashi, Omura (2007); Horton, Ross (2009); Asano, Moroi, Sato, Yanagida (2011); Akula, Liu, Nath, Peim (2011); Feng, Matchev, Sanford (2011); Younkin, Martin (2012); …

FOCUS POINT SUSY

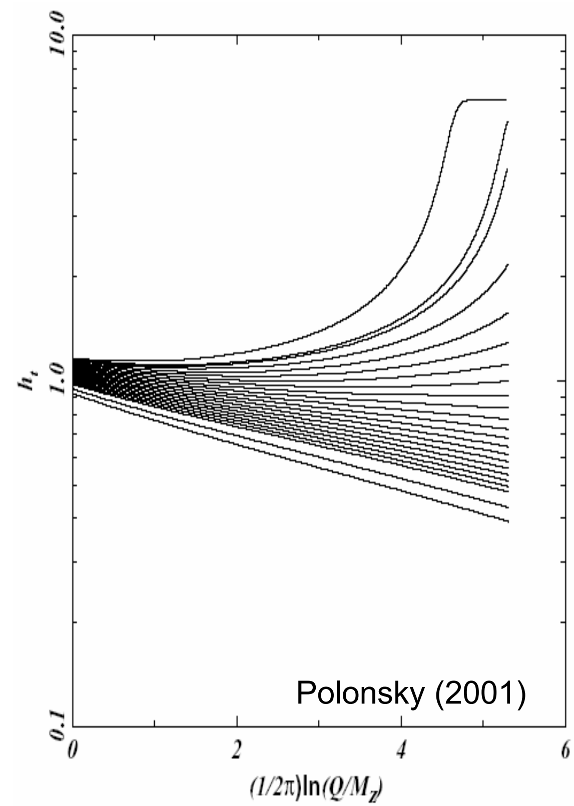
- RGEs play a crucial role in almost all of the main motivations for weak-scale SUSY: coupling constant unification, radiative EWSB, top quark quasi-fixed point. What about naturalness?



Martin (1997)



Olive (2003)



Polonsky (2001)

FP SUSY: ANALYTIC EXPLANATION

- For low and moderate tanβ,

$$\frac{1}{2}m_Z^2 = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$
$$\approx -\mu^2 - m_{H_u}^2$$

- So focus on scalar mass $m_{H_u}^2$

- Scalar masses enter only their own RGEs:

$$\begin{aligned} \dot{g} &\sim g^3 \\ \dot{y} &\sim g^2 y - y^3 \\ \dot{M}_{1/2} &\sim g^2 M_{1/2} \\ \dot{A} &\sim -g^2 M_{1/2} - y^2 A \\ \dot{m}^2 &\sim g^2 M_{1/2}^2 - y^2 A^2 - y^2 m^2 \end{aligned}$$

- Assume A, $M_{1/2}$ << m (natural by $U(1)_R$ symmetry).

- If there is one dominant Yukawa,

$$\dot{\boldsymbol{m}}^2 = -\frac{y^2}{16\pi^2}\boldsymbol{N}\boldsymbol{m}^2$$

and the masses evolve as

$$\boldsymbol{m}^2(0) = \sum_i \kappa_i \boldsymbol{e}_i \rightarrow \boldsymbol{m}^2(t) = \sum_i \kappa_i \boldsymbol{e}_i e^{-\lambda_i \int^t \frac{y^2}{16\pi^2}dt'}$$

where $(\boldsymbol{e}_i, \lambda_i)$ are the eigenvectors and eigenvalues of *N*.

LOW AND MODERATE TANβ

$$\begin{bmatrix} \dot{m}^2_{H_u} \\ \dot{m}^2_{U_3} \\ \dot{m}^2_{Q_3} \end{bmatrix} = -\frac{y_t^2}{16\pi^2} \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} m^2_{H_u} \\ m^2_{U_3} \\ m^2_{Q_3} \end{bmatrix}$$

$$\begin{bmatrix} m^2_{H_u}(m_W) \\ m^2_{U_3}(m_W) \\ m^2_{Q_3}(m_W) \end{bmatrix} = \kappa_1 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} e^{-6\int^{t_W} \frac{y^2}{16\pi^2} dt'} + \kappa_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \kappa_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

- The exponent is very nearly 1/3, and so

$$\begin{bmatrix} m^2_{H_u}(0) \\ m^2_{U_3}(0) \\ m^2_{Q_3}(0) \end{bmatrix} = m_0^2 \begin{bmatrix} 1 \\ 1+x \\ 1-x \end{bmatrix} \rightarrow \begin{bmatrix} m^2_{H_u}(m_W) \\ m^2_{U_3}(m_W) \\ m^2_{Q_3}(m_W) \end{bmatrix} = m_0^2 \begin{bmatrix} 0 \\ \frac{1}{3}+x \\ \frac{2}{3}-x \end{bmatrix}$$

- m_{H_u} evolves to zero for any (even multi-TeV) m_0, and so the weak scale is natural, even though the stops are heavy

HIGH TANβ

$$\begin{bmatrix} \dot{m}^2_{H_u} \\ \dot{m}^2_{U_3} \\ \dot{m}^2_{Q_3} \\ \dot{m}^2_{D_3} \\ \dot{m}^2_{H_d} \end{bmatrix} = -\frac{y_t^2}{16\pi^2} \begin{bmatrix} 3 & 3 & 3 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m^2_{H_u} \\ m^2_{U_3} \\ m^2_{Q_3} \\ m^2_{D_3} \\ m^2_{H_d} \end{bmatrix} - \frac{y_b^2}{16\pi^2} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} m^2_{H_u} \\ m^2_{U_3} \\ m^2_{Q_3} \\ m^2_{D_3} \\ m^2_{H_d} \end{bmatrix}$$

- For $y_t = y_b$, a similar analysis shows that (remarkably)

$$\begin{bmatrix} m^2_{H_u}(0) \\ m^2_{U_3}(0) \\ m^2_{Q_3}(0) \\ m^2_{D_3}(0) \\ m^2_{H_d}(0) \end{bmatrix} = m_0^2 \begin{bmatrix} 1 \\ 1+x \\ 1-x \\ 1+x-x' \\ 1+x' \end{bmatrix}$$

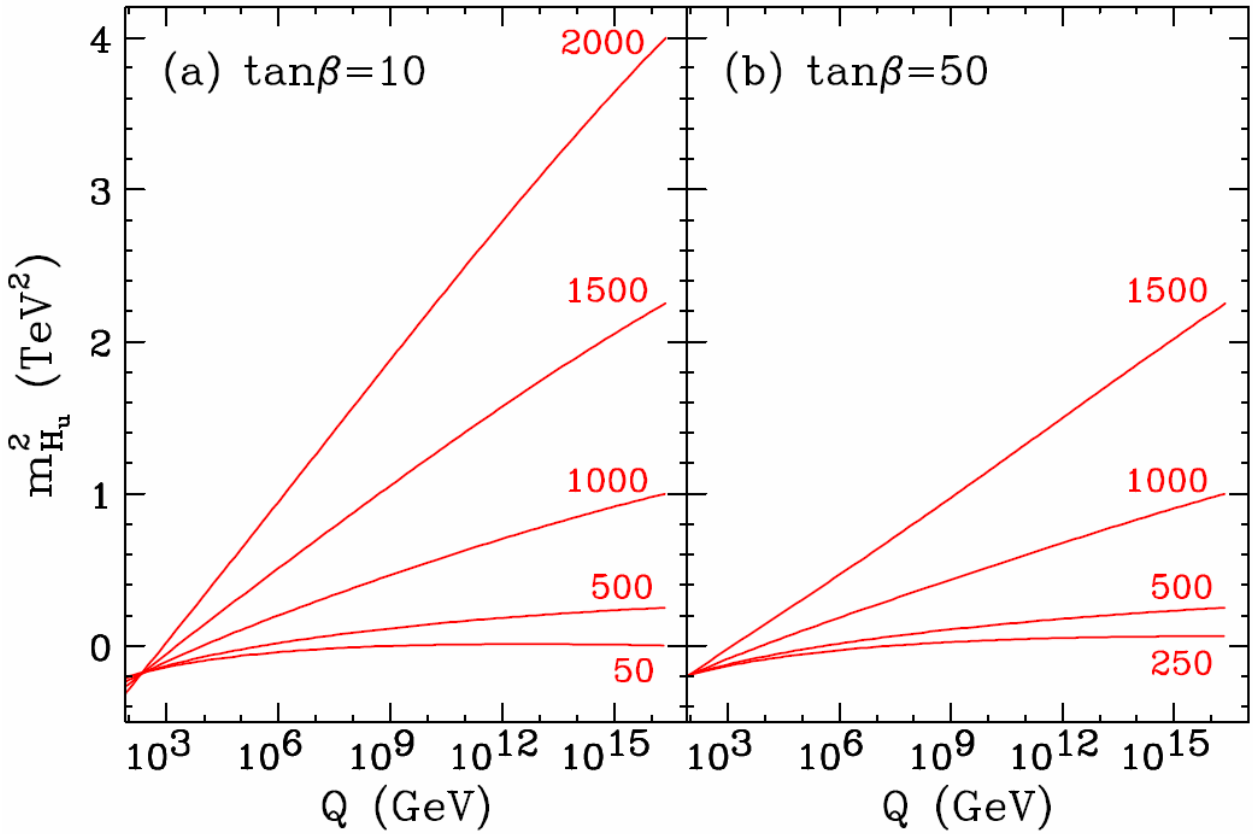
implies $m_{Hu} = 0$ at the weak scale

- SUMMARY: mSUGRA/CMSSM is a special case, but FP SUSY is far more general
  - x and x' are arbitrary
  - All other scalar masses can be anything
  - A, $M_{1,2,3}$ can be anything, provided they are within conventional naturalness limits
  - tanβ can be anything

FP SUSY: GRAPHICAL EXPLANATION

- Families of RGEs have a focus point (cf. fixed point)

- Dynamically-generated hierarchy between the stop masses and the weak scale



- The weak scale is insensitive to variations in the fundamental parameters
- All natural theories with heavy stops are focus point theories

FP SUSY: NUMERICAL EXPLANATION

- By dimensional analysis, can write m_{Hu} in the following form and see the FP numerically:

$$
\begin{aligned}
-2m_{Hu}^2(M_z) \; = \; & 5.45M_3^2 + 0.0677M_3M_1 - 0.00975M_1^2 \\
& +0.470M_2M_3 + 0.0135M_1M_2 - 0.433M_2^2 \\
& +0.773A_tM_3 + 0.168A_tM_2 + 0.0271A_tM_1 \\
& +0.214A_t^2 \boxed{- 1.31m_{Hu}^2 + 0.690m_{Q_3}^2 + 0.690m_{U_3}^2}
\end{aligned}
$$

Abe, Kobayashi, Omura (2007)

- In fact, special cases of FP SUSY can be seen in the results of some early (pre-top quark) studies

Alvarez-Gaume, Polchinski, Wise (1983); Barbieri, Giudice (1988)

- The underlying structure is obscured by the numerical calculations, but this is also a way forward to find new FP possibilities, e.g., involving non-universal gaugino masses

Abe, Kobayashi, Omura (2007); Horton, Ross (2009); Younkin, Martin (2012)

IMPLICATIONS

- Naturalness is useful if it leads us toward theories that describe data. How does a theory with heavy scalars fare?

- FP SUSY has many nice features
  - Higgs boson mass
  - Coupling constant unification and proton decay
  - Natural suppression of EDMs
  - Excellent dark matter candidate (mixed Bino-Higgsino)

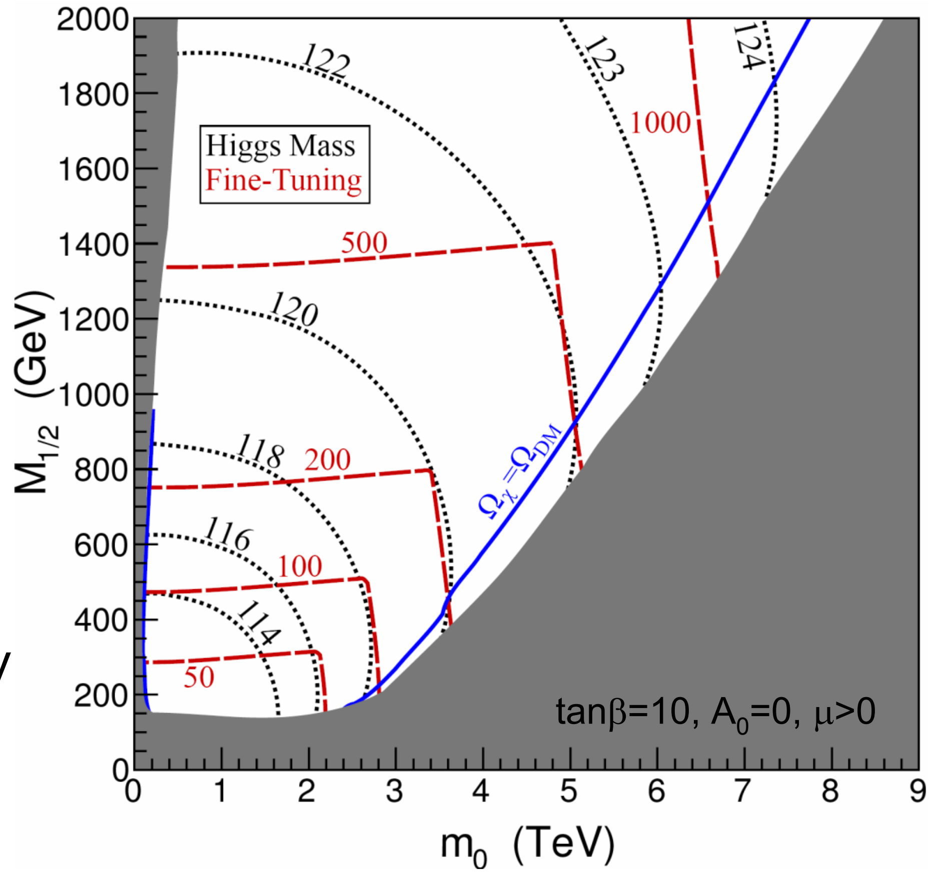
Feng, Matchev (2000); Feng, Matchev, Wilczek (2000)

- Cf. split SUSY: Essentially identical phenomenology motivated by the anthropic principle

Arkani-Hamed, Dimopoulos (2004); Giudice, Romanino (2004)

HIGGS BOSON

- Consider the special case of mSUGRA/CMSSM
- Higgs boson mass in the currently allowed range 115.5 GeV – 127 GeV
- Compatible with hints of Higgs signal
  - CMS 124 GeV, ATLAS 126 GeV
  - Expt. uncertainties ~ 1-2 GeV
  - Theory uncertainties ~ few GeV



Feng, Matchev, Sanford (2011)

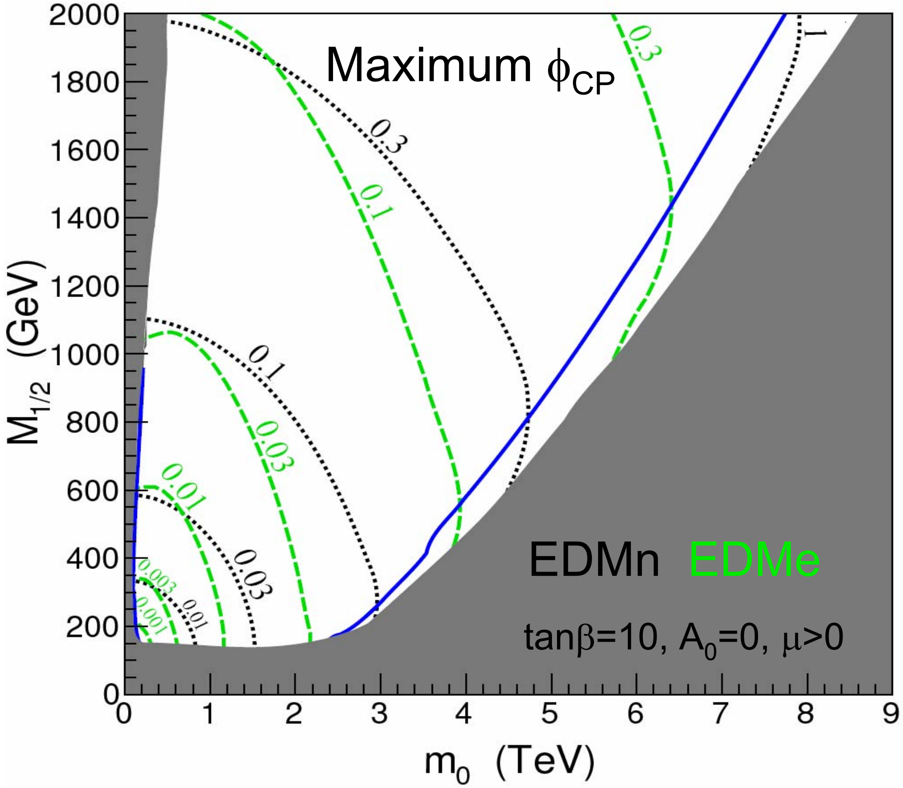
ELECTRIC DIPOLE MOMENTS

- EDMs are flavor-conserving, CP-violating, not eliminated by scalar degeneracy
- Stringent bounds on electron and neutron EDMs

Regan et al. (2002)
Baker et al. (2006)

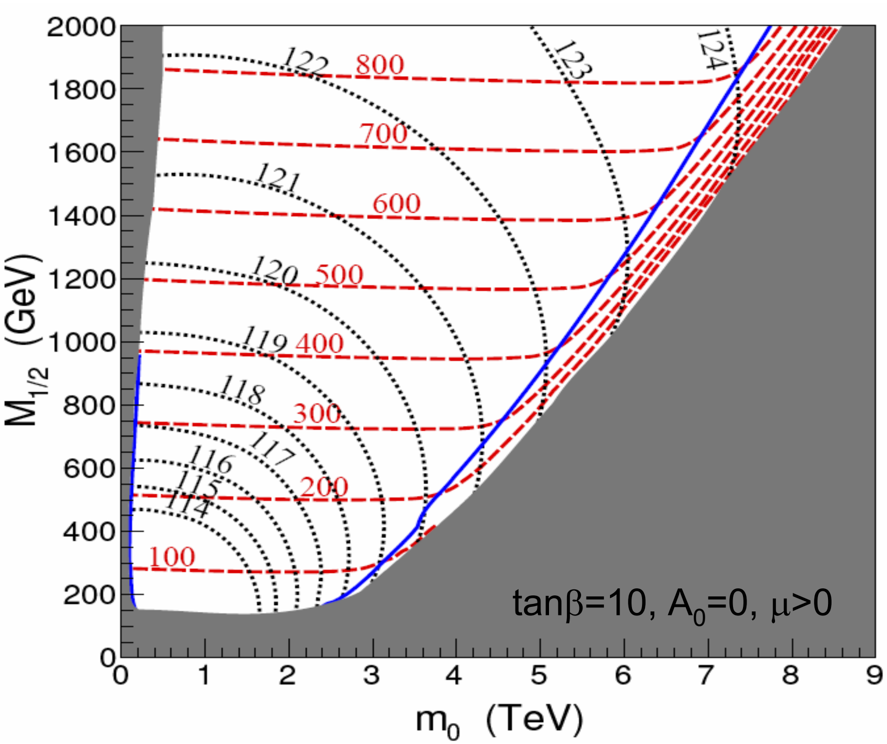
- O(1) phases → multi-TeV scalars
- EDMs naturally satisfied in FP SUSY, but ongoing searches very promising

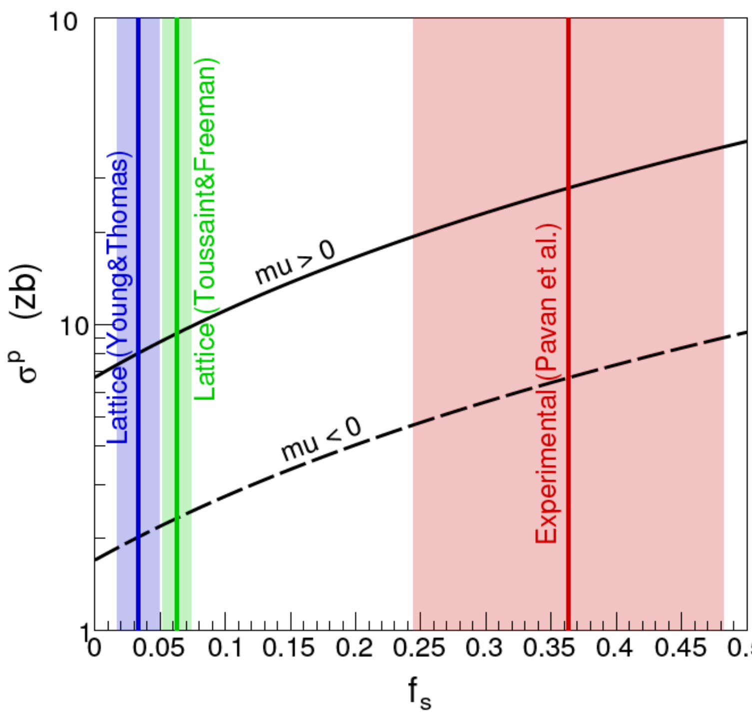
$$d_f = \frac{1}{2} e\, m_f\, g_2^2\, |M_2 \mu|\, \tan\beta\, \sin\phi_{\rm CP}\, K_C(m^2_{\tilde{f}_L}, |\mu|^2, |M_2|^2)$$



Feng, Matchev, Sanford (2011)

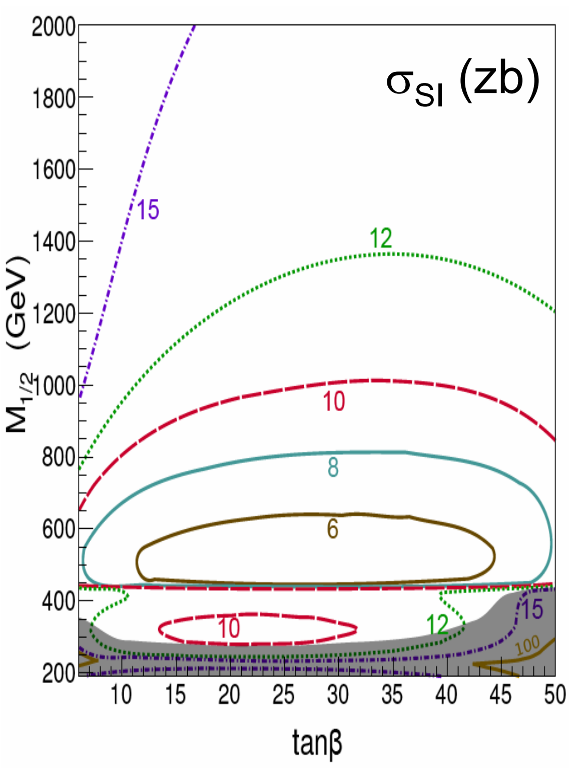
NEUTRALINO DARK MATTER



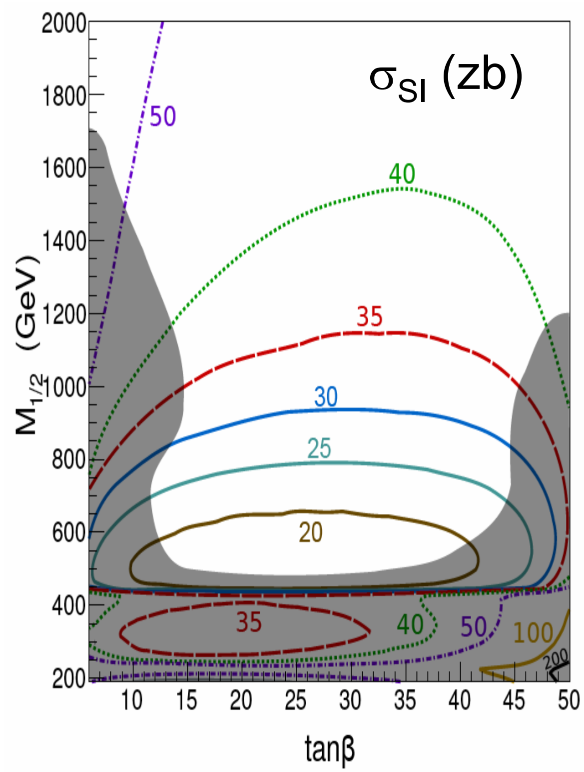


- Masses: ~60 GeV – TeV
- Direct detection cross section: strong dependence on strange content

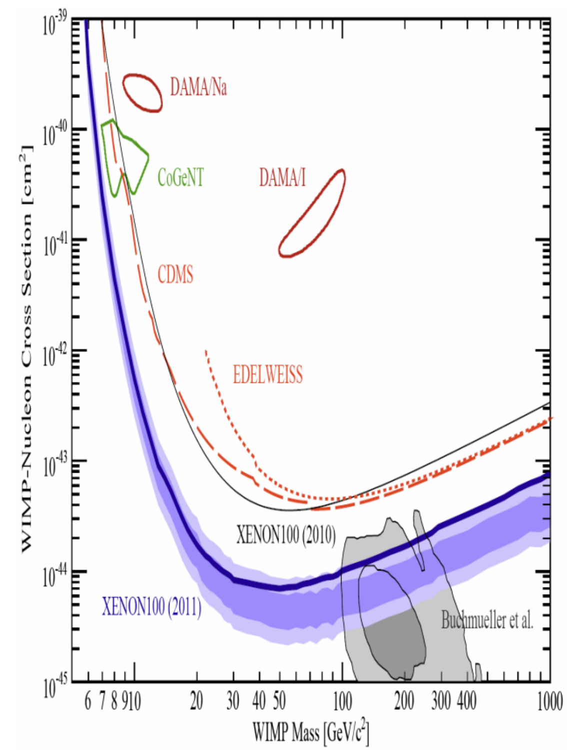
NEUTRALINO DIRECT DETECTION



(a) $f_s = 0.05,\ \mu > 0$



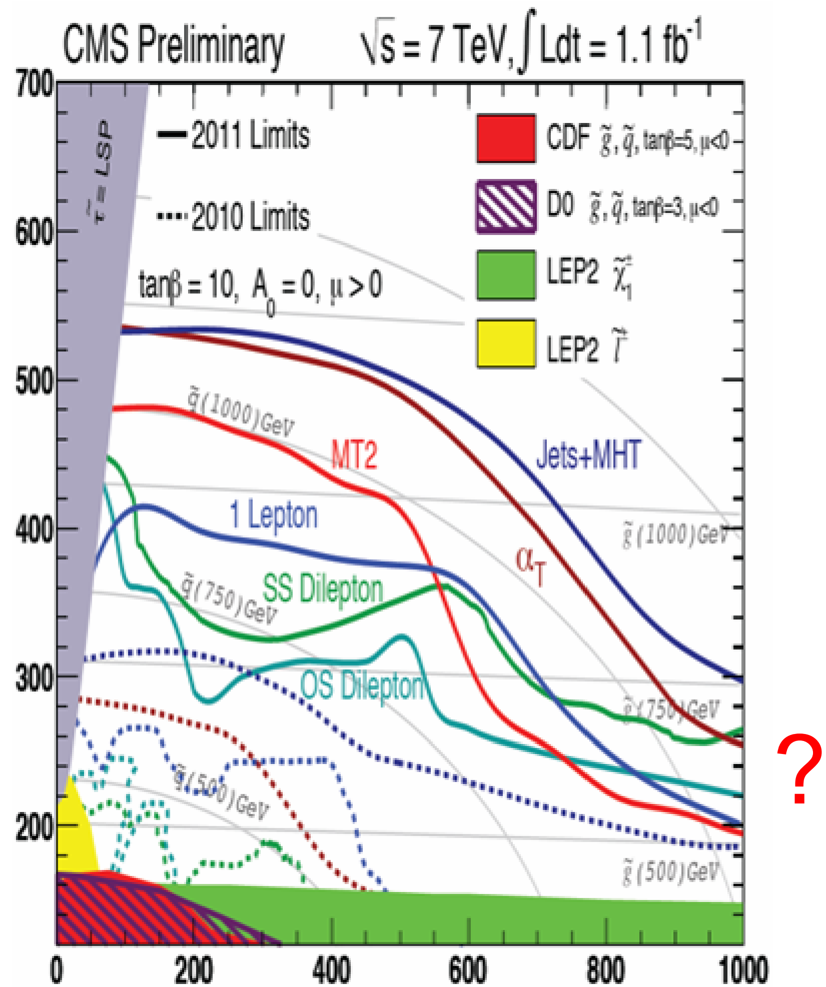
(b) $f_s = 0.36,\ \mu > 0$



- Not excluded, but a signal should be seen in the near future (e.g., XENON at APS April meeting, …)

LHC

- Conventional wisdom: SUSY is in trouble, CMSSM is excluded
- Actually, SUSY is fine, the CMSSM has never been more useful and likely to be (effectively) correct
- Custom-built for analysis: Higgs results, etc. → SUSY is already a simplified model, with just a few parameters (μ, M_1, M_2, M_3, possibly smuons for g-2)
- More attention needed



HIGGS IN GMSB

- The Higgs boson poses a puzzle for SUSY with gauge-mediated SUSY breaking

Draper, Meade, Reece, Shih (2011); Evans, Ibe, Shirai, Yanagida (2012)

- But let's consider the dark matter problem in GMSB

- Neutralino DM is not an option: the original motivation for GMSB is the solution to flavor problems, and this requires $m_{\tilde{G}} < 0.01\ m_\chi$

- keV gravitino DM is also not particularly attractive now:
  $\Omega_{\tilde{G}} h^2 \approx 0.1\ (m_{\tilde{G}}\ /\ 80\ \text{eV})$, but Lyman-$\alpha$ constraints → $m_{\tilde{G}} > 2$ keV.

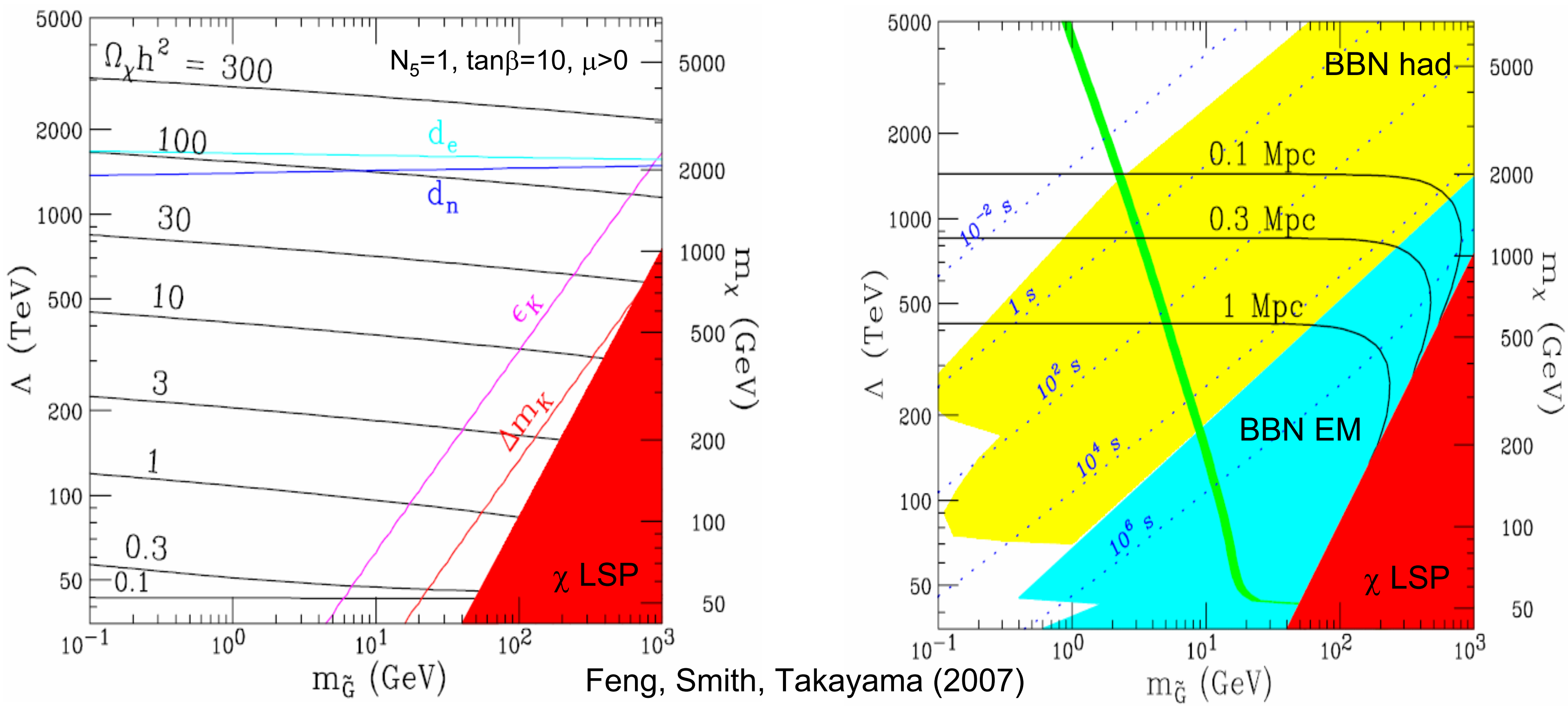
Viel et al. (2006); Seljak et al. (2006)

GOLDILOCKS SUSY

Feng, Smith, Takayama (2007)
Kitano, Low (2005)

- Neutralinos are (over-)produced in the early universe, decay to gravitinos that form DM. Recall: over-producing neutralinos is not hard!
- Why "Goldilocks":
  - Gravitinos are light enough to solve the flavor problem
  - Gravitinos are heavy enough to be all of DM
- $\Omega_\chi \sim m_\chi^2$, $\Omega_{\tilde{G}} \sim m_\chi m_{\tilde{G}}$; flavor → $m_{\tilde{G}} / m_\chi < 0.01$
- Solution guaranteed for sufficiently large m_χ , $m_{\tilde{G}}$
- But is it natural? Consider mGMSB

GOLDILOCKS IN MINIMAL GMSB

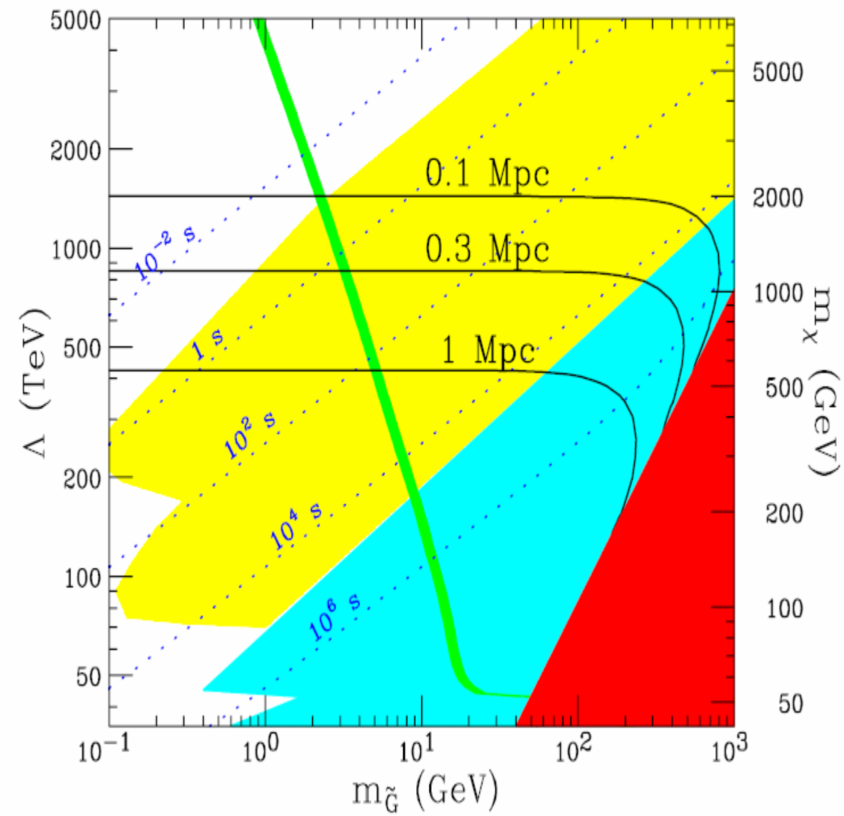


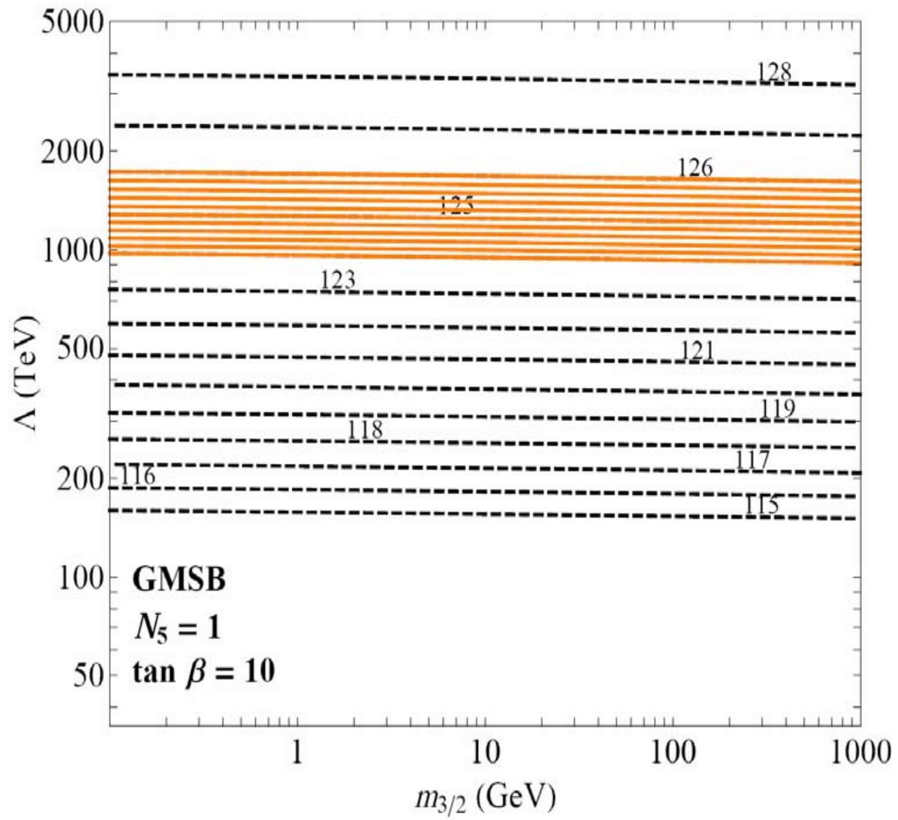
Feng, Smith, Takayama (2007)

- Particle physics: EDMs → multi-TeV superpartners
- Cosmology: Ω_χ ~ 100, m_χ ~ 1 TeV, $m_{\tilde{G}}$ ~ 1 GeV
- Astrophysics: BBN constraints, $\tilde{G}$ DM can't be hot

GOLDILOCKS AND THE HIGGS

Feng, Surujon, Yu (in progress)





- For Goldilocks DM, the preferred region of mGMSB also implies Higgs masses in the preferred range

SUMMARY

- Higgs boson results are changing what SUSY models are allowed, preferred

- Focus Point SUSY: all natural theories with heavy stops are FP theories; reconciles naturalness with Higgs boson mass, fits all data so far; expect DM signal in near future

- Goldilocks SUSY: Higgs results fit beautifully in a scenario with a heavy spectrum and late decays of neutralinos to gravitino DM