• Light Higgs excluded outside $115.5 \text{ GeV} < m_H < 127 \text{ GeV}$
• Hints for Higgs signal in the upper half of this interval
• No strong indications of non-SM Higgs couplings
HIGGS RESULTS AND SUSY

• 30,000 foot view: great for SUSY
• Closer view: challenging for SUSY
  – Higgs mass requires heavy top squarks
  – Naturalness requires light top squarks
• This tension is much more direct that the tension created by bounds on flavor and CP violation
• It has been present (to a lesser degree) since LEP2
OUTLINE

• Naturalness

• Focus Point SUSY (Gravity-Mediated SUSY)
  Feng, Matchev, Sanford (2011, in progress)

• Goldilocks SUSY (Gauge-Mediated SUSY)
  Work with Rajaraman, Takayama, Smith, Cembranos (2003-2007)
  Feng, Surujon, Yu (in progress)
NATURALNESS

• Two approaches:

• Option 1: “I know it when I see it.” Justice Potter Stewart

• Option 2: Quantify with some well-defined naturalness prescription

• Option 1 acknowledges that naturalness is subjective, but is a non-starter. Option 2 provides an opportunity for discussion and insights, as long as its limitations are appreciated.
A NATURALNESS PRESCRIPTION

- Step 1: Choose a framework with input parameters. E.g., mSUGRA with

\[ \{P_{\text{input}}\} = \{m_0, M_{1/2}, A_0, \tan \beta, \text{sign}(\mu)\} \]

- Step 2: Fix all remaining parameters with RGEs, low energy constraints. E.g., at the weak scale, tree-level,

\[ \frac{1}{2} m_Z^2 = \frac{m_{H_u}^2 - m_{H_d}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \]

- Step 3: Choose a set of parameters as free, independent, and fundamental. E.g., mSUGRA with

\[ \{a_i\} = \{m_0, M_{1/2}, A_0, B_0, \mu_0\} \]

- Step 4: Define sensitivity parameters

\[ c_i \equiv \left| \frac{\partial \ln m_Z}{\partial \ln a_i} \right| \]

Ellis, Enqvist, Nanopoulos, Zwirner (1986)
Barbieri, Giudice (1988)

- Step 5: Define the fine-tuning parameter

\[ c = \max \{c_i\} \]
COMMENTS

• Step 1: Choose a framework with input parameters. E.g., mSUGRA with

\[ \{P_{\text{input}}\} = \{m_0, M_{1/2}, A_0, \tan \beta, \text{sign}(\mu)\} \]

This is absolutely crucial. Generic SUSY-breaking is excluded, there must be structure leading to correlated parameters, and the correlations impact naturalness. There is no model-independent measure of naturalness.

• Step 2: Fix all remaining parameters with RGEs, low energy constraints. E.g., at the weak scale

\[ \frac{1}{2} m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2}{\tan^2 \beta - 1} \tan^2 \beta + \mu^2 \]

Important to refine this to include 2-loop RGEs, 1-loop threshold corrections, minimize the potential at some appropriate scale (typically, the geometric mean of stop masses).
• Step 3: Choose a set of parameters as free, independent, and fundamental. E.g., mSUGRA with \( \{a_i\} = \{m_0, M_{1/2}, A_0, B_0, \mu_0\} \)

A popular choice is \( \{a_i\} = \{\mu_0\} \), which leads to \( c = 2\mu^2/m_Z^2 \). This is a simple, but completely deficient and misleading, measure of naturalness.

Should we include other parameters, like \( y_t \)?

– No – Ellis, Enqvist, Nanopoulos, Zwirner (1986); Ciafaloni, Strumia (1996), Bhattacharyya, Romanino (1996); Chan, Chattopadhyay, Nath (1997); Barbieri, Strumia (1998); Giusti, Romanino, Strumia (1998); Chankowski, Ellis, Olechowski, Pokorski (1998); …

– Yes – Barbieri, Giudice (1988); Ross, Roberts (1992); de Carlos, Casas (1993); Anderson, Castano (1994); Romanino, Strumia (1999); …

We favor No – we are trying understand the naturalness of the SUSY explanation of the gauge hierarchy, so include only SUSY breaking parameters. Note: this is not an issue of what is measured and what isn’t: with our current understanding, if \( \mu \) were measured to be 1 EeV \( \pm \) 1 eV, it will be precisely measured, but completely unnatural.
• Step 4: Define sensitivity parameters \( c_i \equiv \left| \frac{\partial \ln m_Z}{\partial \ln a_i} \right| \).

Why not \( c_i \equiv \left| \frac{\partial \ln m_Z}{\partial \ln a_i} \right| \) (original definition) or \( c_i \equiv \left| \frac{\partial \ln m_Z}{\partial \ln a_i^2} \right| \)?

Factors of 2 or 4 are completely insignificant.

• Step 5: Define the fine-tuning parameter \( c = \max \{c_i\} \).

Why not add in quadrature? What if \( c \) is large for all possible parameter choices (cf. \( \Lambda_{QCD} \))?

And finally, what is the maximal natural value for \( c - 10, 100, 1000, \ldots \)? If SUSY reduces \( c \) from \( 10^{32} \) to 1000, isn’t that enough?
GENERAL STRATEGIES

• Hidden Higgs, Buried Higgs: Make $m_h < 115$ GeV compatible with collider constraints
  Dermisek, Gunion (2005); Bellazzini, Csaki, Falkowski, Weiler (2009); …

• Golden region, mirage mediation: Lower the messenger scale to the weak scale, generate large stop mixing
  Kitano, Nomura (2005); Perelstein, Spethmann (2007)…

• Beyond the MSSM (NMSSM,…): Increase particle content to raise $m_h$ naturally, accommodate non-SM Higgs properties
  Hall, Pinner, Ruderman (2011); Ellwanger (2011); Arvanitaki, Villadoro (2011); Gunion, Jiang, Kraml (2011); Perez (2012); King, Muhlleitner, Nevzorov (2012); Kang, Li, Li (2012);…

• Focus Point SUSY: Dynamically generated naturalness
  Feng, Matchev, Moroi (1999); Feng, Matchev, Wilczek (2000); Feng, Matchev (2000); Abe, Kobayashi, Omura (2007); Horton, Ross (2009); Asano, Moroi, Sato, Yanagida (2011); Akula, Liu, Nath, Peim (2011); Feng, Matchev, Sanford (2011); Younkin, Martin (2012); …
• RGEs play a crucial role in almost all of the main motivations for weak-scale SUSY: coupling constant unification, radiative EWSB, top quark quasi-fixed point. What about naturalness?
FP SUSY: ANALYTIC EXPLANATION

- For low and moderate $\tan \beta$,
  \[
  \frac{1}{2} m_Z^2 = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1}
  \approx -\mu^2 - m_{H_u}^2
  \]

- So focus on scalar mass $m_{H_u}^2$

- Scalar masses enter only their own RGEs:
  \[
  \begin{align*}
  \dot{g} & \sim g^3 \\
  \dot{y} & \sim g^2 y - y^3 \\
  \dot{M}_{1/2} & \sim g^2 M_{1/2} \\
  \dot{A} & \sim -g^2 M_{1/2} - y^2 A \\
  \dot{m}^2 & \sim g^2 M_{1/2}^2 - y^2 A^2 - y^2 m^2
  \end{align*}
  \]

- Assume $A, M_{1/2} << m$ (natural by $U(1)_R$ symmetry).

- If there is one dominant Yukawa,
  \[
  \dot{m}^2 = -\frac{y^2}{16\pi^2} N m^2
  \]
  and the masses evolve as
  \[
  m^2(0) = \sum_i \kappa_i e_i \rightarrow m^2(t) = \sum_i \kappa_i e_i e^{-\lambda_i \int \frac{y^2}{16\pi^2} dt'}
  \]
  where $(e_i, \lambda_i)$ are the eigenvectors and eigenvalues of $N$. 

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Feng 12
LOW AND MODERATE TAN$\beta$

$$\begin{bmatrix} \dot{m}^2_{H_u} \\ \dot{m}^2_{U_3} \\ \dot{m}^2_{Q_3} \end{bmatrix} = - \frac{y_t^2}{16\pi^2} \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} m^2_{H_u} \\ m^2_{U_3} \\ m^2_{Q_3} \end{bmatrix}$$

$$\begin{bmatrix} m^2_{H_u}(m_W) \\ m^2_{U_3}(m_W) \\ m^2_{Q_3}(m_W) \end{bmatrix} = \kappa_1 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} e^{-6 \int \frac{y^2}{16\pi^2} dt'} + \kappa_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \kappa_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

- The exponent is very nearly 1/3, and so

$$\begin{bmatrix} m^2_{H_u}(0) \\ m^2_{U_3}(0) \\ m^2_{Q_3}(0) \end{bmatrix} = m_0^2 \begin{bmatrix} 1 \\ 1 + x \\ 1 - x \end{bmatrix} \rightarrow \begin{bmatrix} m^2_{H_u}(m_W) \\ m^2_{U_3}(m_W) \\ m^2_{Q_3}(m_W) \end{bmatrix} = m_0^2 \begin{bmatrix} 1 + x \\ \frac{1}{3} - x \end{bmatrix}$$

- $m_{H_u}$ evolves to zero for any (even multi-TeV) $m_0$, and so the weak scale is natural, even though the stops are heavy.
For $y_t = y_b$, a similar analysis shows that (remarkably)

$$
\begin{bmatrix}
\dot{m}_{H_u}^2 \\
\dot{m}_{U_3}^2 \\
\dot{m}_{Q_3}^2 \\
\dot{m}_{D_3}^2 \\
\dot{m}_{H_d}^2
\end{bmatrix}
= \frac{y_t^2}{16\pi^2}
\begin{bmatrix}
3 & 3 & 3 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
m_{H_u}^2 \\
m_{U_3}^2 \\
m_{Q_3}^2 \\
m_{D_3}^2 \\
m_{H_d}^2
\end{bmatrix}
= \frac{y_b^2}{16\pi^2}
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 2 & 2 & 2 \\
0 & 0 & 3 & 3 & 3
\end{bmatrix}
\begin{bmatrix}
m_{H_u}^2 \\
m_{U_3}^2 \\
m_{Q_3}^2 \\
m_{D_3}^2 \\
m_{H_d}^2
\end{bmatrix}
$$

\begin{align*}
\left[
\begin{array}{c}
m_{H_u}^2(0) \\
m_{U_3}^2(0) \\
m_{Q_3}^2(0) \\
m_{D_3}^2(0) \\
m_{H_d}^2(0)
\end{array}
\right]
&= m_0^2
\begin{bmatrix}
1 \\
1 + x \\
1 - x \\
1 + x - x' \\
1 + x'
\end{bmatrix}
\end{align*}

implies $m_{H_u} = 0$ at the weak scale.

**SUMMARY:** mSUGRA/CMSSM is a special case, but FP SUSY is far more general
- $x$ and $x'$ are arbitrary
- All other scalar masses can be anything
- $A$, $M_{1,2,3}$ can be anything, provided they are within conventional naturalness limits
- $\tan \beta$ can be anything
FP SUSY: GRAPHICAL EXPLANATION

• Families of RGEs have a focus point (cf. fixed point)

• Dynamically-generated hierarchy between the stop masses and the weak scale

• The weak scale is insensitive to variations in the fundamental parameters

• All natural theories with heavy stops are focus point theories
FP SUSY: NUMERICAL EXPLANATION

• By dimensional analysis, can write $m_{Hu}$ in the following form and see the FP numerically:

$$-2m_{Hu}^2(M_z) = 5.45M_3^2 + 0.0677M_3M_1 - 0.00975M_1^2$$
$$+0.470M_2M_3 + 0.0135M_1M_2 - 0.433M_2^2$$
$$+0.773A_tM_3 + 0.168A_tM_2 + 0.0271A_tM_1$$
$$+0.214A_t^2 - 1.31m_{Hu}^2 + 0.690m_{Q_3}^2 + 0.690m_{U_3}^2$$

Abe, Kobayashi, Omura (2007)

• In fact, special cases of FP SUSY can be seen in the results of some early (pre-top quark) studies


• The underlying structure is obscured by the numerical calculations, but this is also a way forward to find new FP possibilities, e.g., involving non-universal gaugino masses

Abe, Kobayashi, Omura (2007); Horton, Ross (2009); Younkin, Martin (2012)
IMPLICATIONS

• Naturalness is useful if it leads us toward theories that describe data. How does a theory with heavy scalars fare?

• FP SUSY has many nice features
  – Higgs boson mass
  – Coupling constant unification and proton decay
  – Natural suppression of EDMs
  – Excellent dark matter candidate (mixed Bino-Higgsino)

  Feng, Matchev (2000); Feng, Matchev, Wilczek (2000)

• Cf. split SUSY: Essentially identical phenomenology motivated by the anthropic principle

HIGGS BOSON

• Consider the special case of mSUGRA/CMSSM

• Higgs boson mass in the currently allowed range 115.5 GeV – 127 GeV

• Compatible with hints of Higgs signal
  – CMS 124 GeV, ATLAS 126 GeV
  – Expt. uncertainties ~ 1-2 GeV
  – Theory uncertainties ~ few GeV

Feng, Matchev, Sanford (2011)
ELECTRIC DIPOLE MOMENTS

- EDMs are flavor-conserving, CP-violating, not eliminated by scalar degeneracy

- Stringent bounds on electron and neutron EDMs

  Regan et al. (2002)
  Baker et al. (2006)

- O(1) phases → multi-TeV scalars

- EDMs naturally satisfied in FP SUSY, but ongoing searches very promising

\[
d_f = \frac{1}{2} e m_f g_2^2 |M_2 \mu| \tan \beta \sin \phi_{\text{CP}} K_C(m_{\tilde{fL}}^2, |\mu|^2, |M_2|^2)
\]
NEUTRALINO DARK MATTER

- Masses: ~60 GeV – TeV
- Direct detection cross section: strong dependence on strange content
NEUTRALINO DIRECT DETECTION

- Not excluded, but a signal should be seen in the near future (e.g., XENON at APS April meeting, ...)

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LHC

- Conventional wisdom: SUSY is in trouble, CMSSM is excluded

- Actually, SUSY is fine, the CMSSM has never been more useful and likely to be (effectively) correct

- Custom-built for analysis: Higgs results, etc. → SUSY is already a simplified model, with just a few parameters ($\mu$, $M_1$, $M_2$, $M_3$, possibly smuons for g-2)

- More attention needed
HIGGS IN GMSB

• The Higgs boson poses a puzzle for SUSY with gauge-mediated SUSY breaking
  Draper, Meade, Reece, Shih (2011); Evans, Ibe, Shirai, Yanagida (2012)

• But let’s consider the dark matter problem in GMSB

• Neutralino DM is not an option: the original motivation for GMSB is the solution to flavor problems, and this requires $m_{\tilde{G}} < 0.01 m_\chi$

• keV gravitino DM is also not particularly attractive now: $\Omega_{\tilde{G}} h^2 \approx 0.1 \left( m_{\tilde{G}} / 80 \text{ eV} \right)$, but Lyman-$\alpha$ constraints $\rightarrow m_{\tilde{G}} > 2 \text{ keV}$.
  Viel et al. (2006); Seljak et al. (2006)
GOLDILOCKS SUSY

Feng, Smith, Takayama (2007)
Kitano, Low (2005)

• Neutralinos are (over-)produced in the early universe, decay to gravitinos that form DM. Recall: over-producing neutralinos is not hard!

• Why “Goldilocks”:
  – Gravitinos are light enough to solve the flavor problem
  – Gravitinos are heavy enough to be all of DM

• \( \Omega_\chi \sim m_\chi^2, \Omega_\tilde{G} \sim m_\chi m_\tilde{G}; \) flavor \( \rightarrow \) \( m_\tilde{G}/m_\chi < 0.01 \)

• Solution guaranteed for sufficiently large \( m_\chi, m_\tilde{G} \)

• But is it natural? Consider mGMSB
GOLDILOCKS IN MINIMAL GMSB

- Particle physics: EDMs $\rightarrow$ multi-TeV superpartners
- Cosmology: $\Omega_\chi \sim 100$, $m_\chi \sim 1$ TeV, $m_{\tilde{G}} \sim 1$ GeV
- Astrophysics: BBN constraints, $\tilde{G}$ DM can’t be hot

18 Mar 09

Feng, Smith, Takayama (2007)
GOLDILOCKS AND THE HIGGS

- For Goldilocks DM, the preferred region of mGMSB also implies Higgs masses in the preferred range
SUMMARY

• Higgs boson results are changing what SUSY models are allowed, preferred

• Focus Point SUSY: all natural theories with heavy stops are FP theories; reconciles naturalness with Higgs boson mass, fits all data so far; expect DM signal in near future

• Goldilocks SUSY: Higgs results fit beautifully in a scenario with a heavy spectrum and late decays of neutralinos to gravitino DM